AN OVERVIEW OF FINITE ELEMENT CONSIDERATIONS FOR COMPUTER MODELING OF MAT FOUNDATIONS

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ABSTRACT: At present, commercially available software is used for mat foundation analysis. Some of these software products are only valid for flat plate modeling and lack a fundamental understanding of mat-soil interaction. The software user should be aware of these facts when using software to analyze mat foundations. There are very few products on the market which are originally developed specifically for mat foundation analysis with an in-depth understanding of mat soil interaction. Mats are relatively thick concrete slabs with typical span to depth ratios varying from as low as 2 to as high as 8. It is reasonable to expect that shear deformation would play an important role in the mat-soil interaction. Therefore, it would be prudent to model a mat foundation using a thick plate formulation rather than a thin plate. RISAFoundation is a newly developed software product based on finite element methods (FEM). RISAFoundation gives the user the option of modeling a mat foundation using thin or thick plates. In this study, different modeling options in finite element analysis (thin plates vs. thick plates vs. solid elements) are discussed and their influence in the mat foundation design is highlighted. The solution using RISAFoundation is compared with a solid element FEM solution. The inclusion of shear deformation in flexural behavior of plates make the plates more flexible, which increases differential settlement and reduces bending moments, thus the overall mat design is impacted significantly. Furthermore, the transverse shear is retrievable in RISAFoundation unlike other software where it is not retrievable.

KEYWORDS: PLATE MODELING: SHEAR DEFORMATION: MAT FOUNDATION: FEM: RISAFOUNDATION: WINKLER SPRING: SOIL PRESSURE

1. INTRODUCTION

A mat foundation is a large concrete slab used to interface one column or more than one column in several lines, with the base soil. It may encompass the entire foundation area or only a portion. A mat foundation may be used where the base soil has a low bearing capacity and/or the column loads are so large that more than 50 percent of the area is covered by conventional spread footings. A common mat foundation configuration is shown in Figure 1. In addition to direct bearing, mat foundations may also be supported by piles. Piles aid in reducing the settlement of a structure built over a highly compressible soil and controlling buoyancy in case the water table is high.

In the past, analyzing mat foundations included two major simplifying assumptions; an infinitely rigid mat and one-way bending [1]. For very stiff mats with fairly uniform column spacing and loads, these assumptions may not introduce serious error. Assuming an infinitely rigid mat allows determination of a bearing pressure distribution by simple statics, ignoring the high pressure concentrations near applied

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loads and the low pressure areas more distant from these loads. The second assumption provides for the mat behavior to be approximated by a one-way, rather than a two-way, bending analysis.

To overcome these limitations, approximation methods were evolved based on the method of finite differences. These approximation methods address the mat flexibility to a limited degree but do not include analyses for two-way bending; these are extremely tedious and cumbersome to use [2].

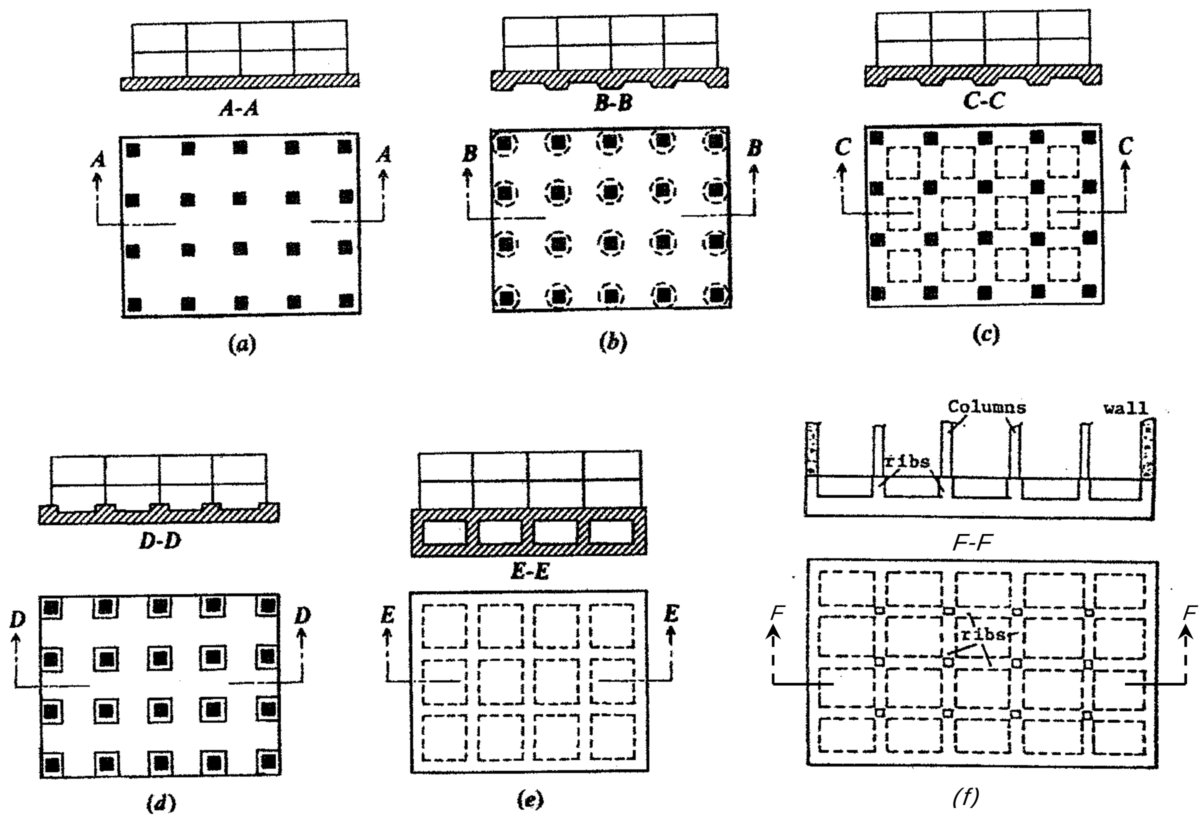

Figure 1. Common types of mat foundations (a) flat plate (b) plates with drop caps (c) waffle slab (d) plate with pedestals (e) cellular slab (f) rib slab

The FEM appears to adequately address mat rigidity and two-way bending and is easily implemented in a general computer code. The FEM is based on the theory of plate bending with the mats supported by the soils. It uses the modulus of sub-grade reaction to designate the soil stiffness and is automatically converted into the compression only nodal springs (called Winkler Springs).

It is common to use a four node quadrilateral plate element to analyze the mat. They are basically 2D elements. The 3D solid modeling of a mat is extremely costly in terms of modeling time and interpreting the results. For this reason, it is not practical to use except for verification purposes.

2. PLATE MODELING OPTION IN RISAFOUNDATION

There are various options in plate modeling, thin, thick, solid etc. The assumption in thin plate modeling is that *"A straight line, normal to the mid surface, remains straight and normal to the deformed mid surface throughout deformation"*.

The first consistent plate theory relying on this assumption has been presented by Kirchhoff [3] in the middle of the nineteenth century. Transverse shear deformations and strains are neglected in these assumptions; the deformation of the entire body is uniquely determined by the displacements of the mid surface. With this assumption, the displacement components of a point of coordinates x, y, and z are;

$$w = w(x, y) \;\; ; \;\; u = -z\beta_x(x, y) \;\; ; \;\; v = -z\beta_y(x, y) \qquad (1)$$

where w is the transverse displacement and β_x and β_y are the rotations of the normal to the undeformed middle surface in the x, z and y, z planes, respectively as shown in Figure 2.

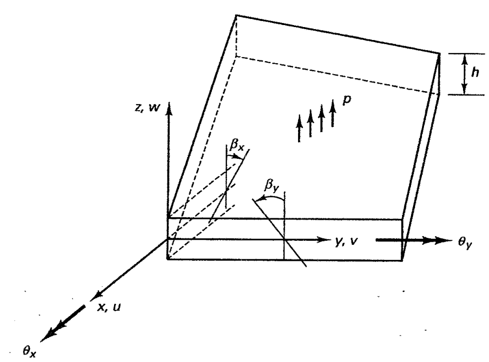

Figure 2. Deformation assumptions in analysis of plate modeling including shear deformations

For thick plate elements (least dimension of plate/thickness smaller than 10), the transverse shear deformation will be significant. Reissner and Mindlin [3] proposed a theory of plates including transverse shear deformation. They used the assumption that *"A straight line, normal to the mid surface, remains straight and but not necessarily normal to the deformed mid surface"*. Considering the plate in Figure 2, the bending strains $\varepsilon_{xx}, \varepsilon_{yy}, \gamma_{xy}$ vary linearly through the plate thickness and are given by the curvatures of the plate, whereas the transverse shear strains are assumed to be constant through the thickness of the plate as follows;

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} = -z \begin{bmatrix} \dfrac{\partial \beta_x}{\partial x} \\ \dfrac{\partial \beta_y}{\partial y} \\ \dfrac{\partial \beta_x}{\partial_y} + \dfrac{\partial \beta_y}{\partial_x} \end{bmatrix} ; \qquad \begin{bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial w}{\partial x} - \beta_x \\ \dfrac{\partial w}{\partial y} - \beta_y \end{bmatrix} \qquad (2)$$

The state of stress in the plate corresponds to plate stress conditions (i.e., $\tau_{zz} = 0$). For an isotropic material, we can thus write

$$\begin{bmatrix} \tau_{xx} \\ \tau_{yy} \\ \tau_{xy} \end{bmatrix} = -z \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \begin{bmatrix} \frac{\partial \beta_x}{\partial x} \\ \frac{\partial \beta_y}{\partial y} \\ \frac{\partial \beta_x}{\partial_y} + \frac{\partial \beta_y}{\partial_x} \end{bmatrix} ; \quad \begin{bmatrix} \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \frac{E}{2(1+\nu)} \begin{bmatrix} \frac{\partial w}{\partial x} - \beta_x \\ \frac{\partial w}{\partial y} - \beta_y \end{bmatrix} \qquad (3)$$

This is the basis of plate modeling in RISAFoundation. RISAFoundation recognizes the fact that the mats are relatively thick and shear deformation must be included [4].

3. MODEL GENERATION

A typical mat is chosen with the following details; 40ft by 40 ft square in plan. It has asymmetric loads as shown in Figure 3. The concrete strength is 4000 psi with a density of 150 pcf. The mat is 40in thick and is meshed using 2ft by 2ft quadrilaterals. Furthermore, the mat has a uniformly distributed load of 40 psf throughout the region. The soil is modeled using an area spring with a subgrade modulus of 300 k/ft3.

The model was analyzed in RISAFoundation. The model was also analyzed using software A and software B (real names intentionally withheld) which both use thin plate element formulations and do not consider the transverse shear deformation.

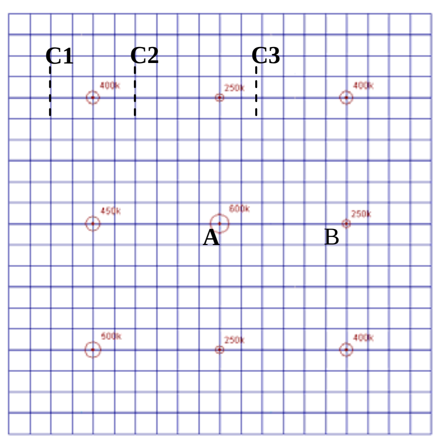

Figure 3. A typical mat foundation model (meshed 2ft by 2ft)

For comparison, we also analyzed the mat using MSC/NASTRAN software. In MSC/NASTRAN, we modeled the mat using a 2ft x 2ft x 0.833 ft eight-node linear element (3D solid element) based on an isoparametric formulation [5].

4. COMPARISON OF RESULTS

The mat deformations obtained from each software are given in Table 1. It is found that compared to thick plate modeling, the deformations in thin plate modeling without transverse shear are underestimated by 7-10%. The deformation obtained by RISAFoundation is higher than that from software A and software B. This is because the inclusion of shear deformation makes the plate more flexible, allowing larger deformations. The deformation in RISAFoundation is more closer to MSC/NASTRAN. A contour plot of soil pressure is shown in Figure 4.

Table 1. Comparison of deflections obtained from software

Software	Deflection (in)	
	At point A	At point B
RISAFoundation	0.140	0.089
MSC/NASTRAN	0.139	0.090
Software A	0.136	0.087
Software B	0.137	0.087

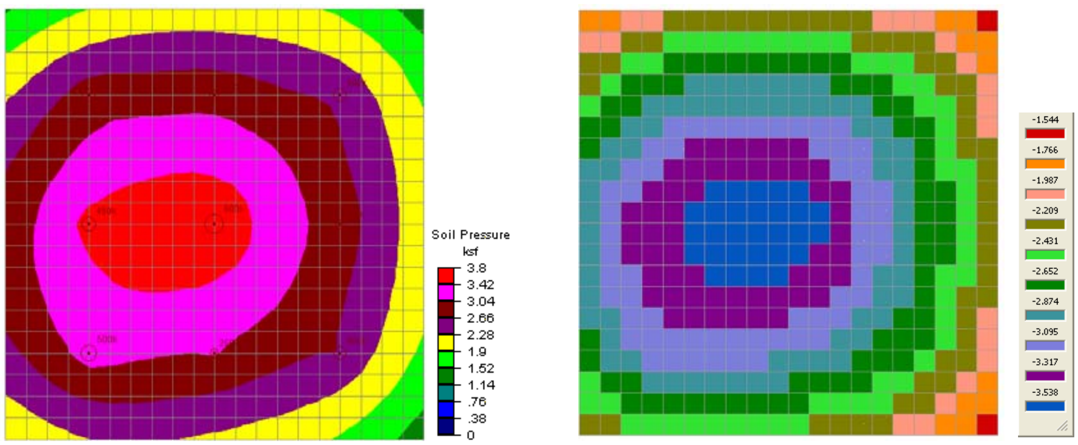

Figure 4. Soil pressure plot (a) RISAFoundation (b) software B

We have also compared the design bending moment and shear. For this, we have selected some design sections in the mat as shown in the Figure 3, namely c1, c2, and c3. We collected total moment and total shear about each cross section which are designated "*design moment*" and "*design shear*". Figure 5 shows the comparison of the design moment and the design shear obtained from all software.

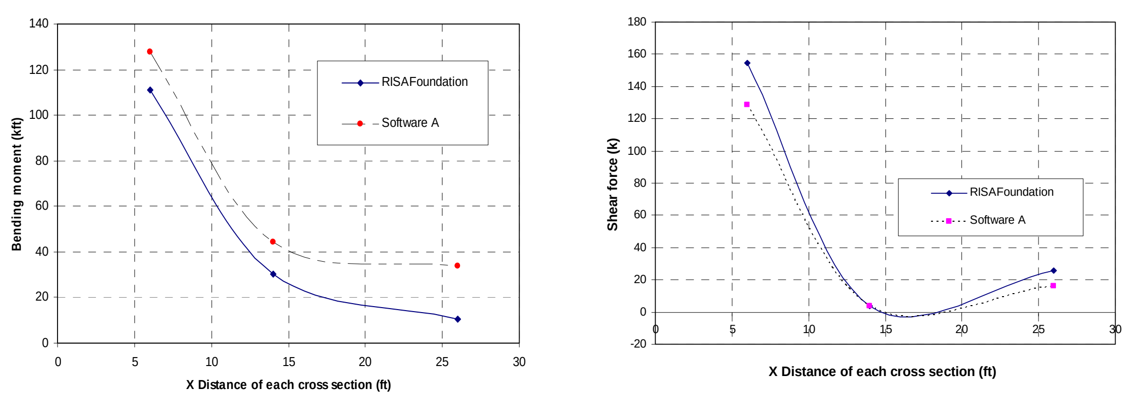

Figure 5. A plot of moments/shears versus design cross sections obtained from various software

It appears that the design moment from RISAFoundation is smaller than that from software A. This is because the inclusion of shear deformation makes the plate flexible. We used plate corner forces to calculate the design moment in RISAFoundation and software A. Software B and MSC/NASTRAN yield a very high moment. This could be because Software B and MSC/NASTRAN give the stresses at the node only, and we have to derive the bending moment by integrating stresses. These stresses are smoothed, averaged and will generally not satisfy the equilibrium within a plate. They are not shown in Figure 5. It is noteworthy to mention that most FEM software is formulated to calculate the maximum surface stresses at a point. In fact, these FEM stresses are of little importance in practical concrete design.

In addition, software A and software B do not give transverse shear, which is a limitation of using thin plate modeling. For comparison purpose, we derived the transverse shear in software A from plate corner forces. The total shear obtained from RISAFoundation is higher than that obtained from software A as shown in Figure 5.

5. CONCLUSIONS

We find that when comparing thin versus thick plate modeling, the thick plate deforms more yielding a smaller design moment. The deformation in RISAFoundation appears closer to MSC/NASTRAN. Therefore, we recommend the use of thick plate modeling for the analysis and design of mat foundations. We also note that the total design moment and shear about a design cross section should be based on corner forces, and not based on FEM stresses. This error may further be amplified where there is a stress discontinuity.

Transverse shear is one of the critical design parameters in mat foundation design which dictates a design strip. The inability to report the transverse shear distribution is one of the limitations in thin plate modeling as observed in software A and software B. Solid 3D solid modeling of mats is very time consuming and is not common in practice.

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