RISA-3D

Rapid Interactive Structural Analysis - 3 Dimensional

Plate Modeling Examples

RISA TECHNOLOGIES

26212 Dimension Drive, Suite 200
Lake Forest, California 92630

(714) 951-5815
(714) 951-5848 (FAX)
WWW.RISATECH.COM

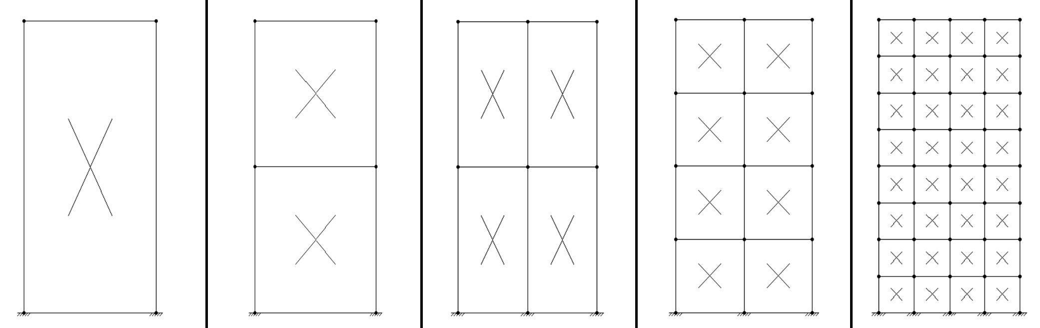
I) Plate Modeling

Plate Model Examples

The following pages are studies of finite element mesh fineness and its relationship to accurate stress and deflection results. These studies are meant to be an aide to help you select an appropriate mesh fineness for a structure you are trying to model. These studies will also answer the "why" many people ask when told they must use a "mesh" of elements to model a structural item (such as a shear wall) instead of using one giant element. Obviously these studies only give an overview of some basic elements and the engineer must be the final judge as to whether a specific finite element model is a good reflection of the "real" structure.

ShearWall Modeling Using Plate/Shell Elements

(Stiffness as a Function of Mesh Fineness)

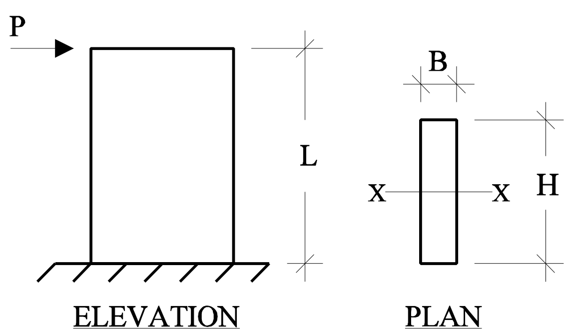


Element Mesh	1 x 1	1 x 2	2 x 2	2 x 4	4 x 8
Deflection (in.)	4.54	8.07	8.26	10.43	11.29
%Theory Defl.	38 %	67 %	69 %	87 %	94 %
K (kips/in)	3304.0	1858.7	1816.0	1438.2	1328.6

Note on Methodology:

Since the theoretical solution is based on an assumption that plane sections shall remain plane after deformation, the last model (4x8 mesh) had very stiff axial members included at the 2nd, 4th, 6th and top level across the width of the wall. This prevented horizontal differential nodel movement and allows for a more meaningful comparison with the theoretical solution.

Theoretical Solution for Shear Wall Deflection due to a Point Load



Shear Wall Sketch

Shear Wall Properties

L = 240 in Area = 1440 in^2
B = 12 in H = 120 in.

E = 4000 ksi $\nu = 0.30$
G = 1538.5 ksi

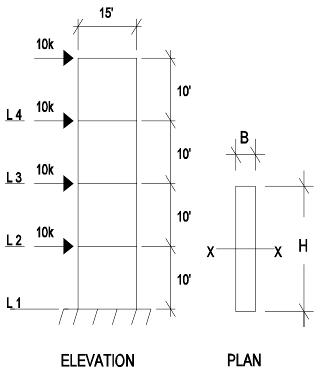
P = 15,000 kip

$I = BH^3/12 = 12(120)^3/12 = 1{,}728{,}000\ in^4$

$\Delta = PL^3/3EI + 1.2PL/AG = 11.95$ in

$K = P/\Delta = 15{,}000k / 11.95in = 1255$ kips/in

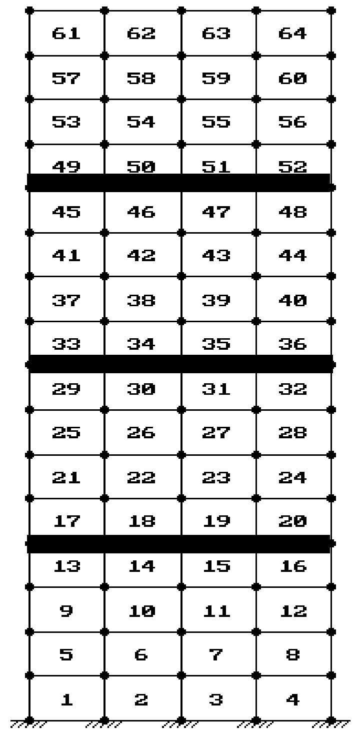
Shear Wall Modeling Using Plate/Shell Elements



Wall Dimensions
H = 15’, B = 6"

Theoretical Results

Flr	Shear	Moment
L4	10 k	1000ft-k
L3	20 k	600 ft-k
L2	30 k	300 ft-k
L1	40 k	100 ft-k



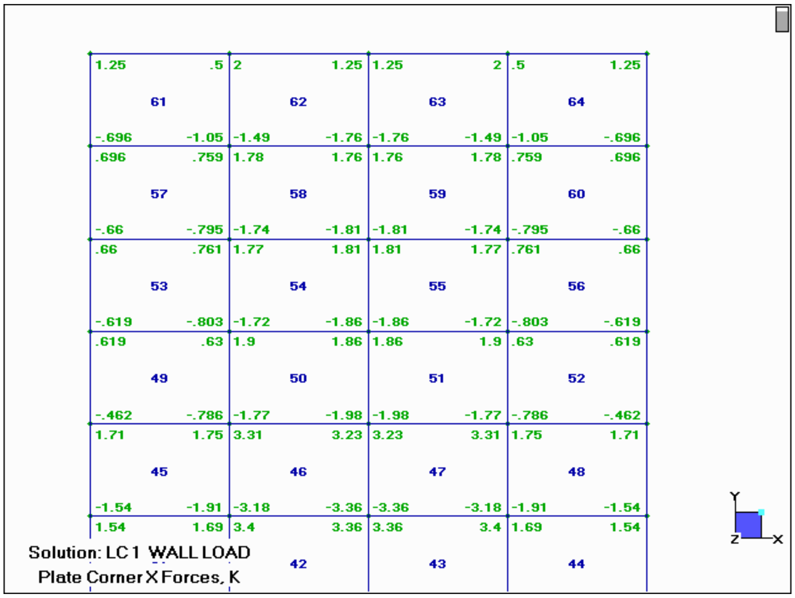
Floor	Shear	Moment	Elements
4	9.99 k	100.05 k’	49-52
3	20 k	300.08 k’	33-36
2	30 k	600 k’	17-20
1	40 k	999.98 k’	1-4

Shown above are the analysis results of a 4 story shear wall. This example is for a straight shear wall, however the method and results are valid for box, channel, or any other shear wall shapes. The RISA3D files that were used to obtain these results are included as “4X1WALL.R3D” and “4X4WALL.R3D”. The 10 kip story loads were applied uniformly across each story. This was done to more accurately model loads being applied to the wall from a rigid or semi-rigid floor. The story shears at each level were calculated as the sum of the FX corner forces. The story moments at each level are calculated from the FY corner forces as shown below :

$$M_i = \left(F_{y_{outer\ node}} \bullet 15ft\right) + \left(F_{y_{inner\ node}} \bullet 7.5ft\right)$$

STORY SHEARS: The story shears were calculated as shown below from the corner forces. See the screen shot close up of the FX corner forces on the next page.

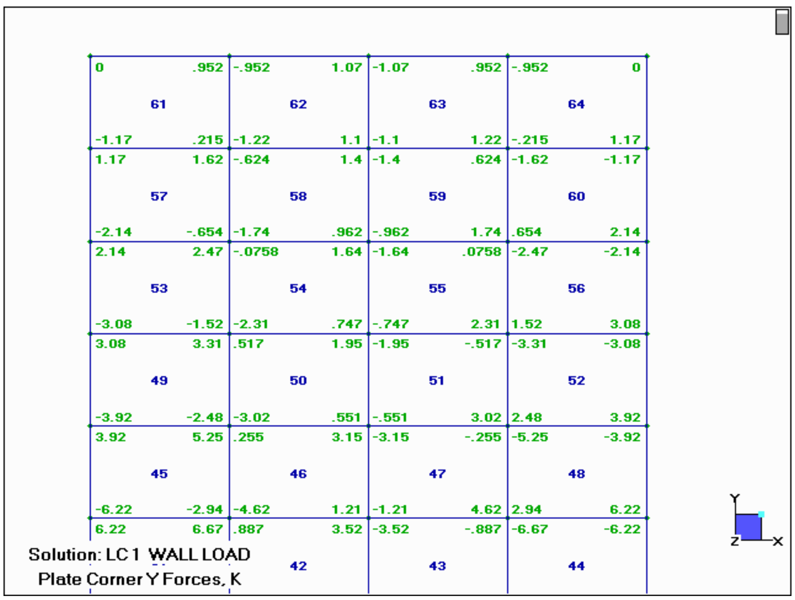
Story	Add Nodal Corner Forces Along the Story Level (kips)	Story Shear
4	[0.462 + 0.786 + 1.77 + 1.98] * 2 = 9.996	9.9 kips
3	[1.2 + 1.65 + 3.45 + 3.7] * 2 = 20.000	20 kips
2	[1.9 + 2.5 + 5.14 + 5.46] * 2 = 30.000	30 kips
1	[11.76 - 0.32 + 6.09 + 2.47] * 2 = 40.000	40 kips



Global Fx Plate Corner Forces - Level 4

For the graphical display of the corner forces, there are 4 corner forces shown for each plate. This is similar to a beam element which has 2 member end forces.

To get the story shear at any line, just sum up all the FX corner forces along the line.



Global Fy Plate Corner Forces - Level 4

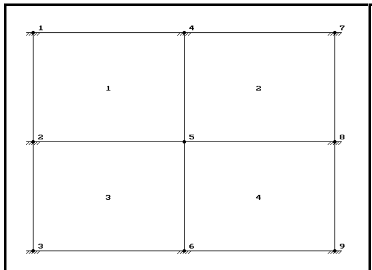
To get the story moment at any line, just sum the moments obtained by multiplying the Fy corner forces along a line, times twice the distance of each Fy force to the center of the wall.

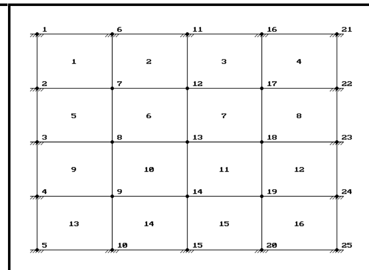
STORY MOMENTS: The story moments were calculated as shown below from the corner forces.

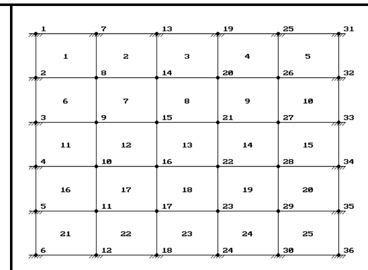
Story	Add the Moments of each Fy force times twice the distance from the center of the wall	Story Moment
4	[3.92 k * 15’ + (2.48 k + 3.02 k) * 7.5’] = 100.05 k’	100.1 kip-ft
3	[12.33 k * 15’ + (7.9 k + 7.45 k) * 7.5’] = 300.075 k’	300.1 kip-ft
2	[24.93 k * 15’ + (16.5 k + 13.64 k) * 7.5’] = 600.00 k’	600 kip-ft
1	[45.81 k * 15’ + (25.16 k + 16.55 k) * 7.5’] = 999.975 k’	999.9 kip-ft

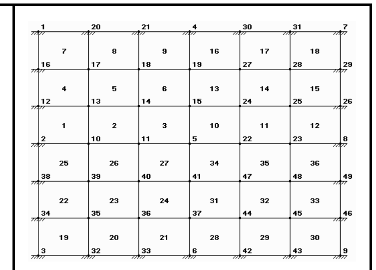
Horizontal Diaphragm Modeling Using Plate/Shell Elements

(Stiffness and Stress Accuracy as a Function of Mesh Fineness)









Element Mesh	2x2	Error from Theory	4x4	Error from Theory	5x5	Error from Theory	6x6	Error from Theory
Deflection @ Center (in)	.032	96%	.895	-1%	.774	13%	.911	-2.2%
M_y @ Center (K-ft / ft)	80.45	19%	75.8	12%	70.3	4%	73.4	8%
Global M_x Reaction @ Center of Long Side to Obtain Max. Local M_y	Reaction at NODE 6		Reaction at NODE 15		Reaction at NODE 18		Reaction at NODE 4	
	844.8 K-ft		717.2 K-ft		545.3 K-ft		489.4 K-ft	
Maximum Local M_y @ Center of Long Side (M_x Reaction divided by the tributary length)	80.5 k-ft /ft	43%	136.6 k-ft /ft	3%	129.8 k-ft /ft	8%	139.8 k-ft /ft	1%

Theoretical Solution For a Plate with Fixed Boundary Conditions

(Results from Roark's Formulas for Stress and Strain, 5th ed., pg. 392)

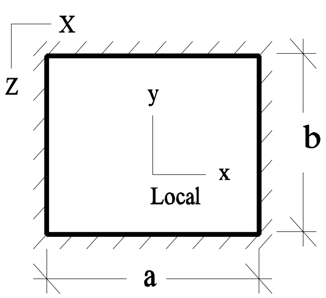


Plate Properties
a = 21 ft
b = 15 ft
thickness = 8 in.
E = 3,122 ksi

Uniform Load
q = 60 psi

For a/b = 1.4
α = 0.0226
β_1 = 0.4356
β_2 = 0.2094

deflection at center (y) = $\alpha q b^4 / E t^3$ = 0.891 in.

$\sigma_{Center} = \beta_2 q b^2 / t^2$ = 6361 psi	$M_{Center} = \sigma_{Center} (t^2/6)$(1kip/1000#) = 67.8 k-ft/ft
$\sigma_{Max} = \beta_1 q b^2 / t^2$ = 13,231 psi	$M_{Max} = \sigma_{Max} (t^2/6)$(1kip/1000#) = 141.1 k-ft/ft

Notes on "Horizontal Diaphragm Modeling with Plate/Shell Elements".

The condition being modeled is a flat plate with fixed edges and a uniform load over the entire surface. The RISA3D files that were used to obtain these results are included as "2X2FIXED.R3D", "4X4FIXED.R3D", "5X5FIXED.R3D", and "6X6FIXED.R3D".

The plate moments at the center of the long side were calculated by dividing the global M_x reaction at the center of the long side by the tributary length. See the summary results below. (Note that the 5x5 mesh produces good results even though the M_x reaction is not at the exact center of the long side.) Remember that plates with perfectly fixed end conditions have their maximum moments at the center edge of their longest side.

Mesh	M_x Global Reaction	Tributary Length	Equation	M_y Local Moment
2x2	844.8 k-ft	21 ft / 2 = 10.5 ft	844.8 / 10.5	80.5 k-ft/ft
4x4	717.2 k-ft	21 ft / 4 = 5.25 ft	717.2 / 5.25	136.6 k-ft/ft
5x5	545.3 k-ft	21 ft / 5 = 4.25 ft	545.3 / 4.25	129.8 k-ft/ft
6x6	489.4 k-ft	21 ft / 6 = 3.5 ft	489.4 / 3.5	139.8 k-ft/ft

The edge moments only need to be considered as the maximum moments when a plate is fixed at it's edges, since the maximum moments will often occur in the center of the plate for most other support conditions. (The edge moments will still need to be considered for moment reversal if the plate is continuous across the supports).

For the situation of continuous slabs supported by beams between columns, the maximum moment will often occur at mid span and not at the edges. Thus a 3x3 or 5x5 mesh should be used to obtain correct moments. Even numbered meshes (e.g. 6x6, 4x4, or 2x2) should be used to obtain the best deflection information and odd numbered meshes (e.g. 3x3 or 5x5) should be used to obtain the best bending moment results. The 6x6 mesh could be used to obtain good moments and deflection results.

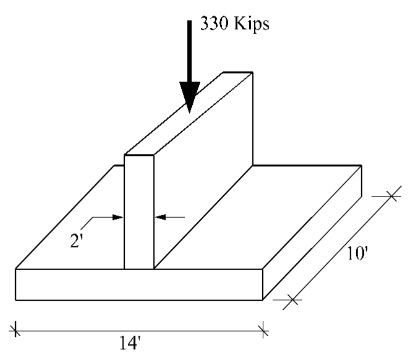
The internal My bending moments are obtained using the Global Corner Forces for the 2x2, 4x4, and 6x6 meshes. The total global MX moment on the side of an element was computed and then divided by the length of the element. (Global MX moments are parallel to local My moments in this model)

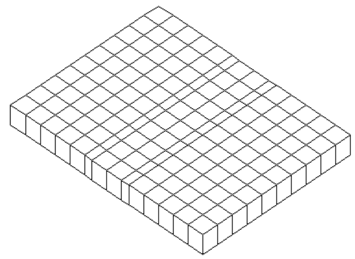
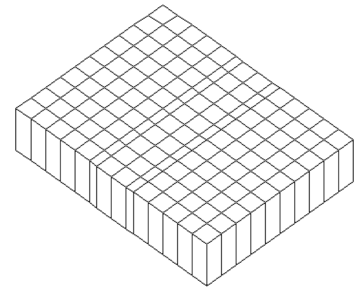
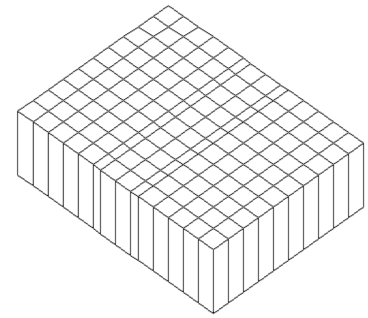
It should be noted that the deflection obtained from the 4x4 and 6x6 meshes is larger than that predicted by the Roark equations because the RISA3D finite element accounts for transverse shear deformation while the Roark equations ignore shear deformation.

These results are for a uniform load. If the loading is more localized, or approaches a point load, a much finer mesh in the vicinity of the load will be needed to model the loading itself and to get accurate results. Also note that RISA3D's finite element (like most commercial finite elements) is based on small strain theory. This means that the in-plane diaphragm stresses are not affected when the transverse deflections become large. According to Roark, (pgs. 405-409), this additional stress becomes significant when the transverse deflection is larger than half the plate thickness.

Spread Footing Modeling Using Plate/Shell Elements

(Stress Accuracy as a Function of Element Distortion)



Footing Thickness	12"	24"	36"
Element Aspect Ratio (Thickness to Length)	1 to 1	2 to 1	3 to 1
One Way Shear at a distance “d” from the face of the wall based on a “flexible” footing (“d” = footing thickness)	109.2 kips	92.4 kips	70 kips
Elements used for Shear results	9 to 135 by 14 and 10 to 136 by 14	10 to 136 by 14 and 11 to 137 by 14	11 to 137 by 14 and 12 to 138 by 14
Moment at the face of the wall based on a “flexible” footing	368 ft-kip	415.2 ft-kip	421.2 ft-kip
Elements use for Moment results	9 to 135 by 14	9 to 135 by 14	9 to 135 by 14
Theoretical One Way Shear at “d” from the face of the wall	117.9 kips	94.3 kips	70.7 kips
Theory Moment at the face of the wall	424.3 ft-kip	424.3 ft-kip	424.3 ft-kip

Shown above are the analysis results for an axial wall load on a spread footing which is then on soil springs. The files used for this parametric study are FLXFTNG.R3D for the “flexible” footing results and RGDFTNG.R3D for the rigid footing results. Note that the theoritical values shown are based on the assumption of an infinitely rigid footing.

As can be seen in the table, the results are converging to the theoretical solution for a infinitely rigid footing as the footing thickness increases and begins to become “very rigid” when compared to the soil spring stiffness.

<u>Notes on Spread Footing Modeling Using Plate/Shell Elements</u>

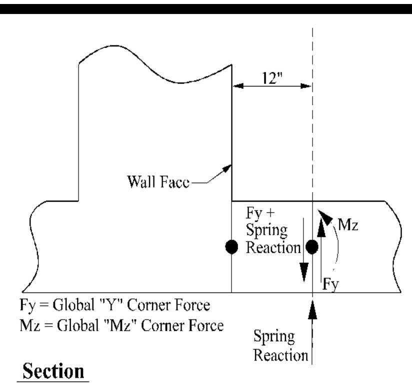
To obtain the One Way Shear values at a distance “d” from the wall face, simply sum the FY global corner force values for the elements on both sides of the appropriate row, and then take the average of these two values. You need to average the two values in this case, because the corner force results are on either side of a soil spring. For example, to obtain the one way shear for the 24” thick footing, sum all the FY corner forces for elements 10 to 136 by 14 (10, 24, 38, ...etc..) and then 11 to 137 by 14 (11, 25, 39,..etc..). Then take the average of those two sums. If you don't have soil springs at the corner force locations, you don't have to average the two values. (The sums on each side in this case will be equal). The easiest way to add up the corner forces is to simply sum them from the graphics display, that way you don't have to worry about the element numbers.

To obtain the Moment values at the face of the wall, just add up the MZ global corner forces for the elements along the wall face. For this example these would be elements 9 to 135 by 14. Again, the easiest way to add up the corner forces is to sum them from the graphics display so you don't have to worry about element numbers.

The finite element corner forces work best when the footing is aligned with the global axes. That way the global corner forces line up with the desired footing shears and moments.

Since the theoretical values for the shear and moment are based on the assumption of an infinitely rigid footing, it is instructive to look at a finite element model where we use an artificially high value of “E” (Elastic Modulus) to approximate an infinitely rigid foundation.

Rigid Footing Results	
Footing Thickness	12"
One Way Shear at “d” from the face of the wall based on a “rigid” footing	117.9 kips
Elements used for Shear	9 to 135 by 14 and 10 to 136 by 14
Moment at the face of the wall based on a “rigid” footing	424.3 ft-kip
Elements use for Moment	9 to 135 by 14



As can be seen, these results agree exactly with the theoretical values.

Computing the Soil Spring Stiffnesses from the Modulus of Subgrade

From a soils engineer, you would obtain the subgrade modulus for a 1' by 1' or .3m. by .3m sample plate. A typical value for medium dense dry sand would be say, k_1 = 500 kcf. This value must first be modified to account for our actual footing size (10 ft by 14 ft). For this example we will use equations from Principles of Foundation Engineering, 2nd edition, by Braja Das, pgs. 240 and 241. We will assume a 1' by 1' sample plate.

$$k_{10'x10'} = k_1\left(\frac{B+1}{2B}\right)^2 = 500\left(\frac{10+1}{2\bullet 10}\right)^2 = 151kcf$$

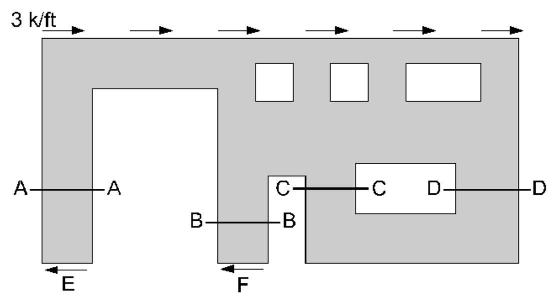
$$k_{10'x13'} = k_{10'x10'}\bullet\left(\frac{1+\frac{B}{L}}{1.5}\right) = 151\left(\frac{1+\frac{10}{14}}{1.5}\right) = 172.57kcf = 0.09987kips/in^3$$

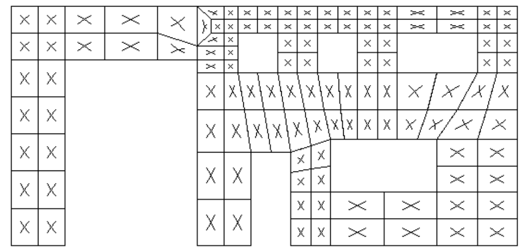
We can now calculate the spring stiffnesses for all the nodes in the model based on tributary area.

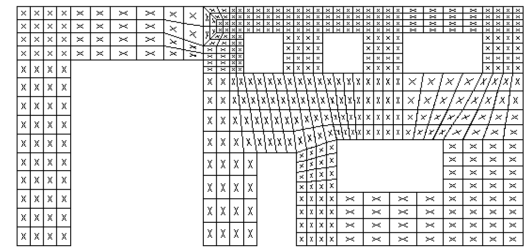
Soil Spring Stiffnesses			
Trib Area	$K_{10` \text{ by } 14'}$ (kips / in^3)	K_{Spring} (kips / in.)	Example Node #
1 ft^2 = 144 in^2	.09987	14.4	13
.5 ft^2 = 72 in^2	.09987	7.2	34
.25 ft^2 = 36 in^2	.09987	3.6	1

A future enhancement to RISA-3D will be the ability to enter a subgrade modulus for a range of elements and then have RISA-3D automatically calculate the soil springs for you based the area tributary to each spring.

Shear Wall Example with Door/Window Penetrations







Horizontal Defl. at Top (in.)	.028 in.	.033 in.
Shear @ A-A	10.82 kips	10.53 kips
Shear @ B-B	24.2 kips	23.08 kips
Shear @ C-C	33.74 kips	33.05 kips
Shear @ D-D	45.26 kips	47.35 kips
Reactions at E	10.8 kips	10.53 kips
Reactions at F	24.2 kips	23.06 kips

This is an example of a typical concrete shear wall with penetrations for windows and doors of various sizes. The files for the models are WALLPEN1.R3D (coarse mesh) and WALLPEN2.R3D (fine mesh). No theoretical solution results are given to compare with, however the two finite element densities are compared to observe the rate of convergence to the “true” answer. The shears at the various lines are computed by adding up the X corner forces for the element corners closest to the lines. The horizontal deflection is for the top of the wall. A very rigid link is added to the top of the wall to simulate the effect of a concrete horizontal diaphragm. This has the effect of stiffening the walls and spreading the load uniformly across the top of the wall. The load is applied as uniform load of 3.0 kips/ft. across the top of the wall. The total width of the wall is 38 ft, so the total applied load is 114 kips. The total height of the wall is 18 ft.

The “coarse” mesh on the left is an example of the minimum finite element mesh that should be used to model this type of wall. Notice that the course mesh gives good results for the wall shears and reactions. The overall deflection of the coarse mesh is off by about 15% from the “fine” mesh. The coarse mesh tends to give too much stiffness to the slender walls around the loading door opening on the left, this can be seen in the larger reactions at points E and F as well in the horizontal deflections. The fine mesh on the right shows that the slender wall sections are more flexible than shown by the coarse mesh and thus the reactions and wall shears are