Appendix A11 Pile Cap Design Calculations

In this example we have a pile cap with 12 HP14x102 piles providing support. The piles have an 85 kip compression capacity, a 12 kip tension capacity and a 14 kip shear capacity. The pile cap is 42" thick with a 6" pile embedment and made from 4 ksi lightweight concrete. A load combination of 1.0*DL+ 1.0*LL is used for the service LC and a load combination of 1.2*DL + 1.6*LL is used for the strength LC.

Geometry, Materials and Criteria

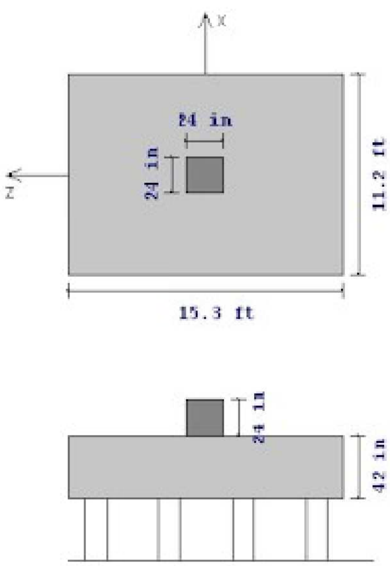
$L_{cap} := 183\ in$ $\quad W_{cap} := 134\ in$ $\quad t_{cap} := 42\ in$ $\quad embed := 6\ in$

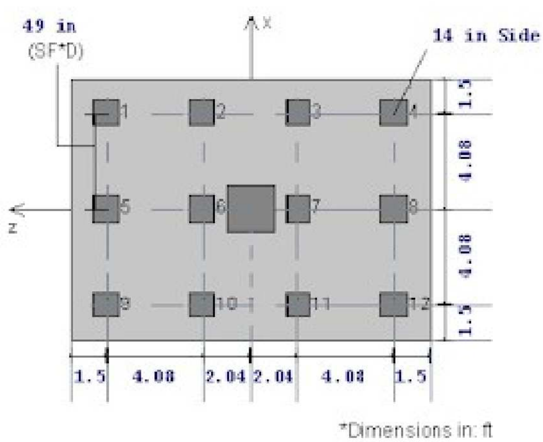
$f_y := 60\ ksi$ $\quad f_c := 4\ ksi$ $\quad \lambda := 0.75$ $\quad \rho_{conc} := .11 \cdot \dfrac{kip}{ft^3}$

$H_{ped} := 24\ in$ $\quad N := 12$ Number of Piles

$L_{ped} := 24\ in$ $\quad d_{pile} := 14\ in$ Side Dimension of Pile

$W_{ped} := 24\ in$ $\quad d_{bar} := 0.75\ in$ Diameter of reinforcement





$l_x := 49\ in$ Distance from c/l of pedestal to c/l of piles in the x direction.

$l_{1z} := 24.5\ in$ Distance from c/l of pedestal to c/l of 1st piles in the z direction.

$l_{2z} := 73.5\ in$ Distance from c/l of pedestal to c/l of 2nd piles in the z direction.

$$w_x := l_x - \frac{W_{ped}}{2} = 37\ in$$

Distance from piles centroid to face of pedestal in x direction.

$$w_{1z} := l_{1z} - \frac{W_{ped}}{2} = 12.5\ in$$

Distance from 1st piles centroid to face of pedestal in z direction.

$$w_{2z} := l_{2z} - \frac{W_{ped}}{2} = 61.5\ in$$

Distance from 2nd piles centroid to face of pedestal in z direction.

Effective Depth Calculations (for bending)

$c := 1.5\ in$ Cover (top and bottom)

$$d := t_{cap} - embed - c - d_{bar} = 33.75\ in$$

Distance from the top of cap to centroid of bottom reinforcement

$$d_{top} := t_{cap} - embed = 36\ in$$

Distance from the top of cap to the top of the piles

Applied Loads

$P_d := 250\ kip$ $V_x := 20\ kip$

$P_l := 350\ kip$ $V_z := 40\ kip$

$$M_x := V_z \cdot \left(H_{ped} + \frac{t_{cap}}{2}\right) = 150\ kip \cdot ft \qquad M_z := V_x \cdot \left(H_{ped} + \frac{t_{cap}}{2}\right) = 75\ kip \cdot ft$$

$$w_{ped} := H_{ped} \cdot L_{ped} \cdot W_{ped} \cdot \rho_{conc} = 0.88\ kip \qquad w_{cap} := L_{cap} \cdot W_{cap} \cdot t_{cap} \cdot \rho_{conc} = 65.562\ kip$$

$$P_{tot} := P_d + P_l + w_{ped} + w_{cap} = 666.442\ kip$$

Pile Forces (Service)

We will assume the individual pile forces are correct and use the RISAFoundation output.

$P_{pile1} := 54.1593\ kip$ $P_{u1} := 76.1068\ kip$ $P_{pile8} := 59.2103\ kip$ $P_{u8} := 84.1884\ kip$

$P_{pile2} := 56.6083\ kip$ $P_{u2} := 80.0252\ kip$ $P_{pile9} := 49.5675\ kip$ $P_{u9} := 68.7599\ kip$

$P_{pile3} := 59.0573\ kip$ $P_{u3} := 83.9435\ kip$ $P_{pile10} := 52.0164\ kip$ $P_{u10} := 72.6782\ kip$

$P_{pile4} := 61.5062\ kip$ $P_{u4} := 87.8619\ kip$ $P_{pile11} := 54.4654\ kip$ $P_{u11} := 76.5966\ kip$

$P_{pile5} := 51.8634\ kip$ $P_{u5} := 72.4333\ kip$ $P_{pile12} := 56.9144\ kip$ $P_{u12} := 80.515\ kip$

$P_{pile6} := 54.3124\ kip$ $P_{u6} := 76.3517\ kip$

$P_{pile7} := 56.7613\ kip$ $P_{u7} := 80.2701\ kip$

Pile Cap Flexural Design

For the flexural design we are simply taking the worst case moment at either face of the pedestal and checking against that. To do this I simply compare the pile forces for each side of the pedesal and take the worst case forces.

$$w_{ucapresistx} := 1.2 \cdot \left(\frac{W_{cap} - W_{ped}}{2}\right) \cdot L_{cap} \cdot t_{cap} \cdot \rho_{conc} = 32.292\ kip$$

$$w_{ucapresistz} := 1.2 \cdot \left(\frac{L_{cap} - L_{ped}}{2}\right) \cdot W_{cap} \cdot t_{cap} \cdot \rho_{conc} = 34.178\ kip$$

$$M_{ux} := \left(P_{u3} + P_{u7} + P_{u11}\right) \cdot w_{1z} + \left(P_{u4} + P_{u8} + P_{u12}\right) \cdot w_{2z} - w_{ucapresistx} \cdot \frac{L_{cap} - L_{ped}}{4}$$

$$M_{ux} = \left(1.438 \cdot 10^{3}\right)\ kip \cdot ft$$

$$M_{uz} := \left(P_{u1} + P_{u2} + P_{u3} + P_{u4}\right) \cdot w_{x} - w_{ucapresistz} \cdot \frac{W_{cap} - W_{ped}}{4} = 932.815\ kip \cdot ft$$

Here are the calculations for minimum steel for both temperature and shrinkage and flexure.

$$A_{sminx} := .0018 \cdot L_{cap} \cdot t_{cap} = 13.835\ in^{2} \qquad A_{sminz} := .0018 \cdot W_{cap} \cdot t_{cap} = 10.13\ in^{2}$$

$$A_{sflexxbot} := \frac{200 \cdot \frac{lbf}{in^{2}} \cdot L_{cap} \cdot d}{f_{y}} = 20.588\ in^{2} \qquad A_{sflexzbot} := \frac{200\ \frac{lbf}{in^{2}} \cdot W_{cap} \cdot d}{f_{y}} = 15.075\ in^{2}$$

$$A_{sreqdxbot} := 6.226 \cdot in^{2}$$

Values given in the program

$$A_{sprovxbot} := 12.812 \cdot in^{2}$$

$$a_{x} := \frac{A_{sprovxbot} \cdot f_{y}}{0.85 \cdot L_{cap} \cdot f_{c}} = 1.235\ in$$

$$PhiMnx := 0.9 \cdot A_{sprovxbot} \cdot f_{y} \cdot \left(d - \frac{a_{x}}{2}\right) = \left(1.91 \cdot 10^{3}\right)\ kip \cdot ft$$

$$UC_{Mx} := \frac{M_{ux}}{PhiMnx} = 0.753$$

$$A_{sreqdzbot} := 9.609\ in^{2}$$

Values given in the program

$$A_{sprovzbot} := 14.137 \cdot in^{2}$$

$$a_z := \frac{A_{sprovzbot} \cdot f_y}{0.85 \cdot W_{cap} \cdot f_c} = 1.862\ in$$

$$PhiMnz := 0.9 \cdot A_{sprovzbot} \cdot f_y \cdot \left(d - \frac{a_z}{2}\right) = \left(2.088 \cdot 10^3\right)\ kip \cdot ft$$

$$UC_{Mz} := \frac{M_{uz}}{PhiMnx} = 0.488$$

In the x direction the Asreqd (and even 4/3 Asreqd) is less than the minimum temperature and shrinkage steel, the program uses that minimum.

In the z direction the 4/3*Asreq'd is greater than the As S&T, thus we use 9.609*4/3 = 12.812 in ^2.

Pedestal Punching Shear Check

$d_b := d = 33.75\ in$ Effective depth of slab for pedestal punching.

$L_1 := W_{ped} + d_b = 57.75\ in$

Side dimensions for the shear perimeter.

$L_2 := L_{ped} + d_b = 57.75\ in$

$$P_{upileped} := P_{u1} + P_{u2} + P_{u3} + P_{u4} + P_{u5} + P_{u8} + P_{u9} + P_{u10} + P_{u11} + P_{u12} = 783.109\ kip$$

This value represents the sum of the factored axial forces in piles outside of the pedestal punching shear perimeter.

$$w_{ucapped} := 1.2 \cdot \left(W_{cap} \cdot L_{cap} - L_1 \cdot L_2\right) \cdot t_{cap} \cdot \rho_{conc} = 67.975\ kip$$

This is the self-weight of the pile cap that is outside of the pedestal punching shear perimeter.

$$P_{upunch} := P_{upileped} - w_{ucapped} = 715.134\ kip$$

$M_{uxped} := 1.6 \cdot M_x = 240\ kip \cdot ft$

Force in the pedestal.

$M_{uzped} := 1.6 \cdot M_z = 120\ kip \cdot ft$

$b_o := 2 \cdot \left(L_1 + L_2\right) = 231\ in$ Punching shear perimeter.

$c_1 := \frac{L_1}{2} = 28.875\ in$ This is the distance from centroid to extreme fiber.

$$A_c := b_o \cdot d_b = \left(7.796 \cdot 10^3\right)\ in^2$$

Ac is the perimeter area of the shear cone.

$$J_c := \frac{d_b \cdot \left(W_{ped} + d_b\right)^3}{6} + \frac{\left(W_{ped} + d_b\right) \cdot d_b^{\ 3}}{6} + \frac{d_b \cdot \left(L_{ped} + d_b\right) \cdot \left(L_{ped} + d_b\right)^2}{2} = \left(4.704 \cdot 10^6\right)\ in^4$$

Jc is the polar moment of inertia and this equation can be found in the commentary to section 11.11.7.2 of the ACI 318-11.

$$\gamma := 0.4$$

$$v_{umax} := \frac{P_{upunch}}{A_c} + \frac{\gamma \cdot M_{uxped} \cdot c_1}{J_c} + \frac{\gamma \cdot M_{uzped} \cdot c_1}{J_c} = 0.102\ ksi$$

This is the critical punching shear stress, combining the axial and moment forces transmitted through the pedestal. Punching equations can be found in the commentary to section 11.11.7.2 of the ACI 318-11. Note that here we are combining the stresses due to the moments to get the worst case stress at a corner of the pedestal punching shear perimeter.

$$f_c := 4000 \cdot \frac{lbf^2}{in^4}$$

$$PhiV_{cpunch} := 0.75 \cdot 4 \cdot \sqrt{f_c} \cdot b_o \cdot d_b = \left(1.479 \cdot 10^3\right)\ kip$$

$$PhiV_{ny} := \frac{\lambda \cdot PhiV_{cpunch}}{b_o \cdot d_b} = 0.142\ ksi$$

$$Punch_{codecheck} := \frac{v_{umax}}{PhiV_{ny}} = 0.719$$

Pile Punching Shear Check

Here we will do a punching shear check for pile 4, the worst case one. The program looks at each pile and calculates a punching shear perimeter for Interior, Edge and Corner scenarios and chooses the smallest value for the check.

For round piles, we calculate an equivalent square dimension such that the perimeter of both are equal.

$d_{pile} = 14\ in$ $\qquad$ $d_{toppunch} := t_{cap} - embed = 36\ in$

$$L_{pile} := 11 \cdot in + d_{pile} + \frac{d_{toppunch}}{2} = 43\ in$$

Because there is no top reinforcement in the pile cap, the slab is considered unreinforced for pile punching. Because of this our Phi factor is now 0.55 and we essentially take 2/3 of the original strength (thus 4 goes to 8/3). The ratio of 2/3*(0.55/0.75) is 0.4888. In the program we use a blanket 50% reduction.

$\phi := 0.55$ $\qquad$ $b_{o1} := 2 \cdot L_{pile} = 86\ in$

$PhiVc_{punch} := \phi \cdot \lambda \cdot \frac{8}{3} \cdot \sqrt{f_c} \cdot b_{o1} \cdot d_{top} = 215.389\ kip$ $\qquad$ If we were to calculate it exactly.

$$PhiVc_{punch2} := \frac{0.75 \cdot \lambda \cdot 4 \cdot \sqrt{f_c} \cdot b_{o1} \cdot d_{top}}{2} = 220.284\ kip$$

$$PhiVc_{punch\_RISA} := \frac{\phi \cdot \lambda \cdot 4 \cdot \sqrt{f_c} \cdot b_{o1} \cdot d_{top}}{2} = 161.542\ kip$$

This is the value the program reports.

$P_{u4} = 87.862\ kip$

$$Pu_{ratio} := \frac{P_{u4}}{PhiVc_{punch2}} = 0.399$$

One Way Shear Check

$w_{1z} = 12.5\ in$ $\qquad$ $w_x = 37\ in$

$d = 33.75\ in$ $\qquad$ $d = 33.75\ in$

Because in the x direction w > d, the critical location is at a distance d from the pedestal. This means that we need to calculate the weight of the pile cap resisting the shear at this location.

$$w_{ucapresistxshear} := \left(\frac{W_{cap} - W_{ped}}{2} - d\right) \cdot L_{cap} \cdot t_{cap} \cdot \rho_{conc} = 10.397\ kip$$

$$V_{ux} := P_{u1} + P_{u2} + P_{u3} + P_{u4} - w_{ucapresistxshear} = 317.54\ kip$$

Because in the z direction w < d, the critical location is at the face of the pedestal.
Because of this we can use the wucapresistz that we used for the moment calculation.

$$V_{uz} := P_{u3} + P_{u4} + P_{u7} + P_{u8} + P_{u11} + P_{u12} - w_{ucapresistz} = 459.197\ kip$$

$M_{ux} = \left(1.438 \cdot 10^3\right)\ kip \cdot ft$ $\qquad$ $M_{uz} = 932.815\ kip \cdot ft$

dz > wz, therefore the critical location for shear at the face of the pedestal.

$As_{providedz} := 12.8122 \cdot in^2$ $\qquad$ $As_{providedx} := A_{sminx} = 13.835\ in^2$

$\rho_{provz} := \frac{As_{providedz}}{W_{cap} \cdot d} = 0.002833$ $\qquad$ $\rho_{provx} := \frac{As_{providedx}}{L_{cap} \cdot d} = 0.00224$

Shear strength in the x direction

dz > wz, therefore the critical location for shear at the face of the pedestal and CRSI Design Handbook equation 13-2 on P.13-26 is used .

$$\frac{M_{ux}}{V_{uz} \cdot d} = 1.114$$

Mu/Vu*d must be less than or equal to 1.0, so use 1.0.

$$MVratio := 1$$

$$v_{cx} := \left(\frac{d}{w_{1z}}\right) \cdot (3.5 - 2.5 \cdot MVratio) \cdot \left(1.9 \cdot \lambda \cdot \sqrt{f_c} + 2500 \frac{lbf}{in^2} \cdot \rho_{provz} \cdot MVratio\right) = 262.46 \ psi$$

$$v_{cmax} := 10 \cdot \sqrt{f_c} = 632.456 \ psi$$

$$Vc\_x := v_{cx} \cdot W_{cap} \cdot d = \left(1.187 \cdot 10^3\right) \ kip$$

Shear strength in the z direction

dz < wz, therefore the critical location for shear is at a distance d from the face of the pedestal and ACI 318-11 Equation 11-5 is used.

$$\frac{M_{uz}}{V_{ux} \cdot d} = 1.044$$

Mu/Vu*d must be less than or equal to 1.0, so use 1.0.

$$MVratio := 1$$

$$v_{cz1} := 1.9 \cdot \lambda \cdot \sqrt{f_c} + 2500 \cdot \frac{lbf}{in^2} \cdot \rho_{provx} \cdot MVratio = 95.725 \ psi$$

$$Vc\_z1 := v_{cz1} \cdot L_{cap} \cdot d = 591.221 \ kip$$

Pedestal Design

Inputs

$d_{pedlongbar} := 1\ in$ $\qquad d_{pedshearbar} := 0.5\ in$ $\qquad cover_{ped} := 1.5\ in$ $\qquad W_{ped} = 24\ in$

$$d_{ped} := W_{ped} - cover_{ped} - d_{pedshearbar} - \frac{d_{pedlongbar}}{2} = 21.5\ in \qquad \lambda = 0.75$$

Concrete Shear Capacity

$$V_{cped} := 2 \cdot \lambda \cdot \sqrt{f_c} \cdot W_{ped} \cdot d_{ped} = 48.952\ kip$$

Steel Shear Capacity

$$A_{sv} := 2 \cdot \left(\frac{d_{pedshearbar}^{\ 2} \cdot \pi}{4}\right) = 0.393\ in^2 \qquad s_{pedshear} := 10\ in$$

$$V_{s1} := \frac{A_{sv} \cdot f_y \cdot d_{ped}}{s_{pedshear}} = 50.658\ kip \qquad V_{smax} := 8 \cdot \sqrt{f_c} \cdot d_{ped} \cdot W_{ped} = 261.078\ kip$$

$$V_s := min\left(V_{s1}, V_{smax}\right) = 50.658\ kip$$

Combined Bending and Axial Forces

$$P_{uped} := 1.2 \cdot P_d + 1.6 \cdot P_l + 1.2 \cdot H_{ped} \cdot W_{ped} \cdot L_{ped} \cdot \rho_{conc} = 861.056\ kip$$

$$M_{uxp} := 1.6 \cdot V_z \cdot H_{ped} = 128\ kip \cdot ft \qquad M_{uvp} := 1.6 \cdot V_x \cdot H_{ped} = 64\ kip \cdot ft$$

For this pedestal the interaction diagram actually produces a worst case code check at the top of the pedestal. Thus, the axial force in the pedestal is not including the pedestal self-weight and the moment at the top is zero in both directions.