

# **RISA-3D**

**Rapid Interactive Structural Analysis – 3 Dimensional**

Verification Problems



RISA Tech, Inc  
27442 Portola Pkwy, Suite 200  
Foothill Ranch, California 92610

(949) 951-5815  
(949) 951-5848 (FAX)

[www.risa.com](http://www.risa.com)

© 2025 RISA Tech, Inc. All Rights Reserved. RISA is part of the [Nemetschek Group](#).

No portion of the contents of this publication may be reproduced or transmitted in any means without the express written permission of RISA.

We have done our best to ensure that the material found in this publication is both useful and accurate. However, please be aware that errors may exist in this publication, and that RISA makes no guarantees concerning accuracy of the information found here or in the use to which it may be put.

# Table of Contents

---

Verification Overview .....	1
Verification Problem 1: Truss Model Axial Forces Comparison .....	2
Verification Problem 2: Cantilever Deflection .....	4
Verification Problem 3: Hot Rolled Steel Frame Member Loads .....	6
Verification Problem 4: Cantilever with Thermal Loads .....	8
Verification Problem 5: Hot Rolled Steel Design Calculations .....	10
Verification Problem 6: Curved Member Forces .....	46
Verification Problem 7: Beam Frequency .....	48
Verification Problem 8: Plate Deflections .....	50
Verification Problem 9: Dynamic (Response Spectra) Analysis .....	53
Verification Problem 10: Wood Design Calculations .....	56
Verification Problem 11: Tapered Hot Rolled Steel Frame Design .....	73
Verification Problem 12: P-Delta Analysis .....	78
Verification Problem 13: Projected Loads .....	84
Verification Problem 14: Solid Elements Comparison .....	87
Verification Problem 15: AISC 16 <sup>th</sup> Edition Tension Members .....	89
Verification Problem 16: AISC 16 <sup>th</sup> Edition Compression Members .....	91
Verification Problem 17: AISC 16 <sup>th</sup> Edition Bending Members .....	93
Verification Problem 18: AISC 16 <sup>th</sup> Edition Shear Members .....	95
Verification Problem 19: AISC 16 <sup>th</sup> Edition Combined Forces and Torsion .....	97
Verification Problem 20: Aluminum Compression Members .....	99
Verification Problem 21: Aluminum Bending Members .....	103
Verification Problem 22: Reinforced Concrete Beam in Bending .....	105

# Verification Overview

---

## Verification Methods

We at RISA maintain a library of dozens of test problems used to validate the computational aspects of RISA programs. In this verification package we present a representative sample of these test problems for your review.

These test problems should not necessarily be used as design examples; in some cases the input and assumptions we use in the test problems may not match what a design engineer would do in a “real world” application. The input for these test problems was formulated to test RISA-3D’s performance, not necessarily to show how certain structures should be modeled.

The RISA-3D solutions for each of these problems are compared to either hand calculations or solutions from other well established programs. By “well established” we mean programs that have been in general use for many years, such as the Berkeley SAPIV program. The original SAPIV program is still the basis for several commercial programs currently on the market (but not RISA-3D).

The reasoning is if two or more independently developed programs that use theoretically sound solution methods arrive at the same results for the same problem, those results are correct. The likelihood that both programs will give the same wrong answers is considered extremely remote.

If discrepancies occur between the RISA-3D and the SAPIV results during testing, we don’t automatically assume SAPIV is correct. Additional testing and hand calculations are used to verify which solution (if either) is correct. There are instances where SAPIV results have been proven to be incorrect.

The data for each of these verification problems is provided. The files are Verification Problem 1.r3d for problem 1, Verification Problem 2.r3d for problem 2, etc. When you install RISA-3D these data files are copied into the ...\**RISA User Data\%USERNAME%\Model Files\Examples** directory. If you want to run any of these problems yourself, just read in the appropriate data file and have at it.

## ***RISA-2D Verification***

Due to the similarities in the two programs, this document can also be used to verify RISA-2D. Therefore, we have created RISA-2D model files (.r2d files) for each two-dimensional verification problem and have included them in the ...\**RISA User Data\%USERNAME%\Model Files\Examples** folder of your RISA-2D installation.

## ***Verification Version***

This document contains problems that have been verified in RISA-3D version 23 and RISA-2D version 21.

# Verification Problem 1

---

## Problem Statement

This problem is a typical truss model (please see Figure 1.1 below). The members are pinned at both ends, thus they behave as truss elements. This particular problem is presented as example 3.7 on page 171 of *Structural Analysis and Design* by Ketter, Lee, and Prawel. The text lists “Q” as the load magnitude and “a” as the panel width. For this solution “Q” is taken as 10 kN and “a” is taken as 2 meters (standard metric units).

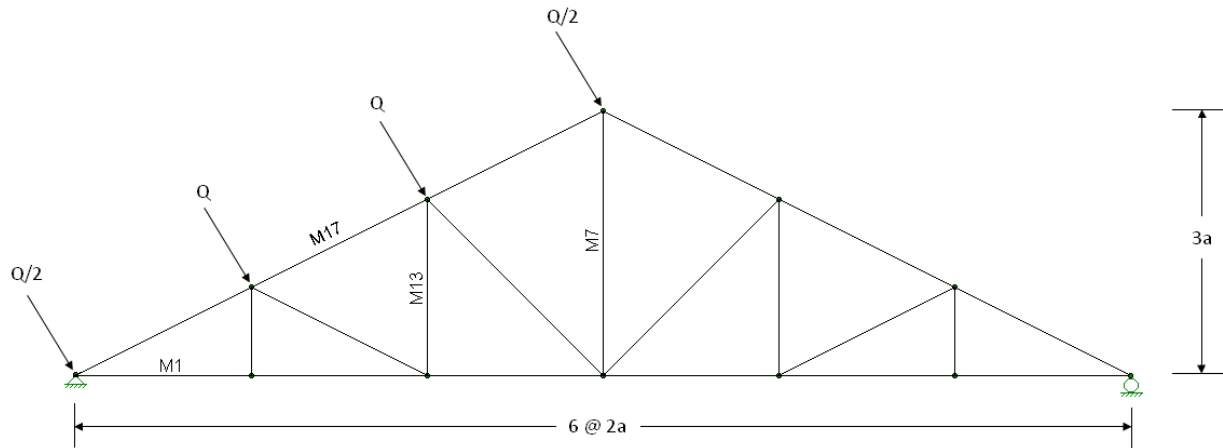


Figure 1.1- Truss Model

This problem provides a comparison of the stiffness method used in RISA-3D with the joint equilibrium method used in the text. The joint equilibrium method may be used to solve statically determinate structures only, while the stiffness method can solve either determinate or indeterminate models.

## Validation Method

The model was created in RISA-3D using W10x17 steel shapes pinned at both ends. The end supports were traditional pin and roller constraints. After solution, the axial force results calculated by RISA-3D are then compared with axial force results presented in the text.

## Comparison

Axial Force Comparison (All Forces in kN)			
Member	RISA-3D	Text	% Difference
M1	39.131	39.131	0.00
M7	11.180	11.180	0.00
M13	5.590	5.590	0.00
M17	-23.750	-23.750	0.00

Table 1.1 – Force Comparison

As seen above, the results match exactly.

**Note:** The text lists tension as positive and compression as negative, opposite of RISA-3D's sign convention. Therefore the signs of the RISA results have been adjusted to match.

# Verification Problem 2

---

## Problem Statement

This model is simply a cantilever with a vertical load applied at the end. The cantilever is 2499 feet in length, modeled using a series of 2499 general section beams, each 1 ft in length (see Figure 2.1). This problem tests the numerical accuracy of RISA-3D. Any significant precision errors would show up dramatically in a model like this.

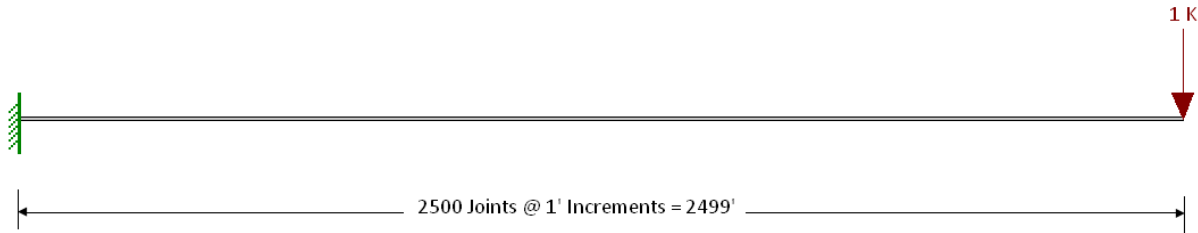


Figure 2.1 – Cantilever Model

## Validation Method

The RISA-3D solution will be compared with the theoretical displacement and rotation for a cantilever with a load at its end (see Table 2.1). The equations are:

Displacement:

$$\Delta = \frac{P * L^3}{3 * E * I}$$

Rotation:

$$\theta = \frac{P * L^2}{2 * E * I}$$

For this model, the following values were used:

$$P = -1 \text{ K}$$

$$L = 2499' (29988'')$$

$$E = 100,000 \text{ ksi}$$

$$A = 10 \text{ in}^2$$

$$I = 10,000 \text{ in}^4$$

$$J = 1 \text{ in}^4$$

Therefore the theoretical solution values are:

$$\Delta = -8989.2 \text{ inches}$$

$$\theta = -0.44964 \text{ radians}$$

## Comparison

Cantilever Solution Comparison (Standard Skyline Solver)			
Value	RISA-3D	Theoretical	% Difference
Displacement (in)	-8989.29	-8989.2	0.001
Rotation (rad)	-0.4496	-0.44964	0.009
Cantilever Solution Comparison (Sparse Accelerated Solver)			
Value	RISA-3D	Theoretical	% Difference
Displacement (in)	-8989.28	-8989.2	0.001
Rotation (rad)	-0.4496	-0.44964	0.009

Table 2.1 – Results Comparison

## Conclusion

As seen above, the results match exactly or have negligible difference.



# Verification Problem 3

---

## Problem Statement

This model is a small 3D frame with oblique members (see Figure 3.1). The purpose of this model is to test RISA-3D's handling of member loads. The members in this model are loaded with full distributed loads, partial length distributed loads, point loads, joint loads, and moments in various load combinations.

In some cases, the loads are used to test RISA-3D against itself. For example, the self-weight capability will also be tested by calculating a set of distributed loads equivalent to the member's self-weight. The solution for these applied loads is compared to the RISA-3D automatic self-weight calculation.

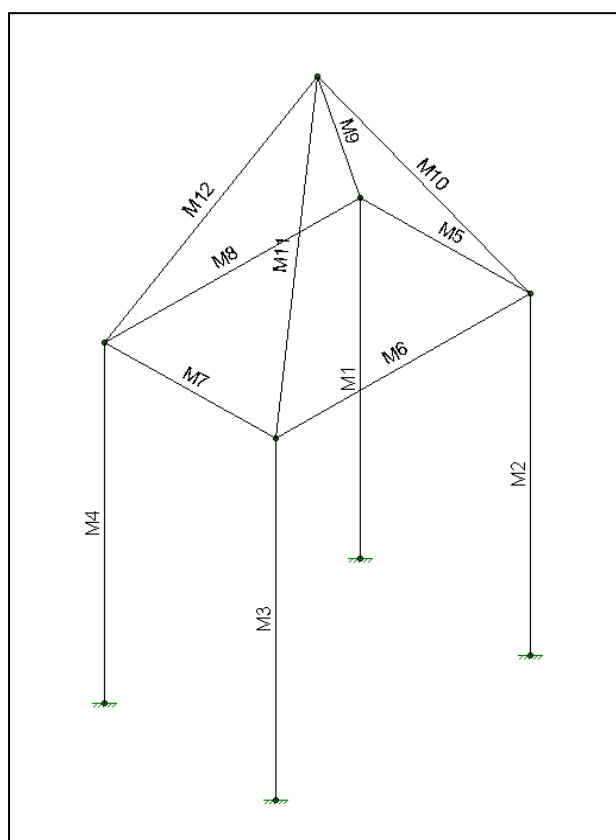


Figure 3.1 – Frame Model

## Validation Method

The RISA-3D results are compared with the solution of this model using the Berkeley SAPIV program (see Table 3.1). SAPIV has been used widely in various forms for well over 20 years. Many commercial programs currently on the market can be traced back to the original SAPIV program.

## Comparison

Member Force Comparison: RISA-3D vs. SAPIV					
Member	Load Combination	Force	RISA-3D	SAPIV	% Difference
M1	7	Axial (k)	8.878	*	0.056
M1	8	Axial (k)	8.883	*	0.056
M9	3	Axial (k)	-17.359	-17.350	0.052
M9	5	Mz (k-ft)	-10.151	-10.150	0.010
M9	6	My (k-ft)	7.535	7.530	0.066
M10	2	Mz (k-ft)	18.606	18.610	0.021
M10	6	Mz (k-ft)	-31.711	-31.700	0.035
M11	1	Mz (k-ft)	-10.690	-10.690	0.000
M11	5	My (k-ft)	2.460	2.450	0.407
M11	6	Z- Shear (k)	-7.799	-7.800	0.013
M12	4	My (k-ft)	4.477	4.480	0.067
M12	5	Y-Shear (k)	3.880	3.880	0.000

Table 3.1 – Force Comparison

\*These results are those in which RISA-3D tested against itself. Load Case 7 is the self-weight defined as applied loads. Load Case 8 is the automatic self-weight calculation, so compare Load Case 7 results to those of Load Case 8.

## Conclusion

As can be seen above, the results match very closely. Any slight variations in the results can be attributed to round off differences.

# Verification Problem 4

---

## Problem Statement

This model is used to test the thermal force calculations in RISA-3D. The model is a five member cantilever with a spring in the local x direction at the free end (see Fig. 4.1). As the model is loaded thermally the spring resist some, but not all, of the thermal expansion.

Thermal loads cause structural behavior somewhat different from other loads. For gravity loads, displacements induce stress; but for thermal loading, displacements cause stress to be relieved. For example, a free end cantilever that undergoes a thermal loading would expand without resistance and thus no stress. Conversely, a fixed-fixed member that undergoes the same thermal loading would see a stress increase with no displacements.

This model uses a spring to provide partial resistance to the thermal load. This is realistic in that members generally would have only partial resistance to thermal effects.

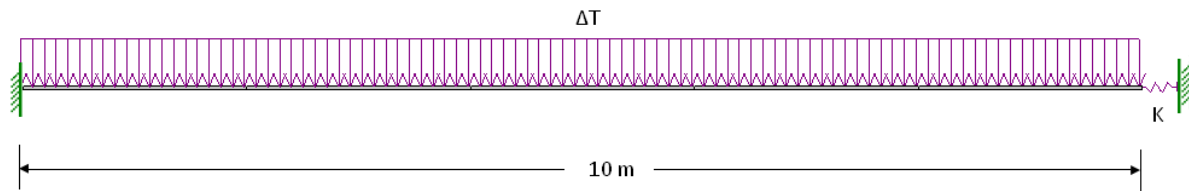


Figure 4.1 – Thermal Model

## Validation Method

The model is validated by the use of hand calculations (see Table 4.1). The theoretically exact solution may be calculated for comparison with the RISA-3D result. Following are those calculations:

### Property Values:

Area (A)	= 50 cm <sup>2</sup>
Young's Modulus (E)	= 70,000 MPa
Thermal Load ( $\Delta T$ )	= 300°
Coefficient of Thermal Expansion ( $\alpha$ )	= 0.000012 cm/cm°C
Spring Stiffness (K)	= 500 kN/cm
Length (L)	= 10 meters

The unrestrained thermal expansion ( $\Delta_{Free}$ ) is:

$$\Delta_{Free} = \alpha * \Delta T * L$$

The general equation for the displacement of a member due to an axial load ( $\Delta_{Axial}$ ) is:

$$\Delta_{Axial} = \frac{P * L}{A * E}$$

We will call the actual displacement of the member “ $\Delta_{Actual}$ .” Now we’ll say “P” is the force in the spring, therefore:

$$P = \Delta_{Actual} * K$$

So, using these formulations, the following is true:

$$\Delta_{Actual} * \frac{K * L}{A * E} = \Delta_{Free} * -\Delta_{Actual}$$

In other words, the “resisted expansion” of the member is the thermal expansion that is not allowed to occur because of the spring and is equal to  $\Delta_{Free} * -\Delta_{Actual}$ . Think of it as the spring force pushing the member end back this resisted expansion distance.

This leads to the equation for the actual displacement:

$$\Delta_{Actual} = \frac{\alpha * \Delta T * L}{1 + \frac{K * L}{A * E}}$$

The force in the member is:

$$Force = \frac{(\Delta_{Free} * -\Delta_{Actual}) * A * E}{L}$$

So for the given property values,

$$\Delta_{Actual} = 1.482 \text{ cm}$$

$$Force = 741.2 \text{ kN}$$

## Comparison

Thermal Results Comparison		
Solution Method	Displacement (cm)	Axial Force (kN)
Exact	1.482	741.20
RISA-3D	1.482	741.18

Table 4.1 – Results Comparison

## Conclusion

As can be seen above, the results match exactly.

# Verification Problem 5

---

## Problem Statement

This verification model is a two bay, two story space frame. The model is comprised of WF, Tee, Channel, and Tube members (see Fig. 5.1). Note the use of the inactive code “Exclude” to isolate only those members to be checked.

This problem is used to verify the stress and steel code check calculations in RISA-3D. Both ASD and LRFD codes will be checked.

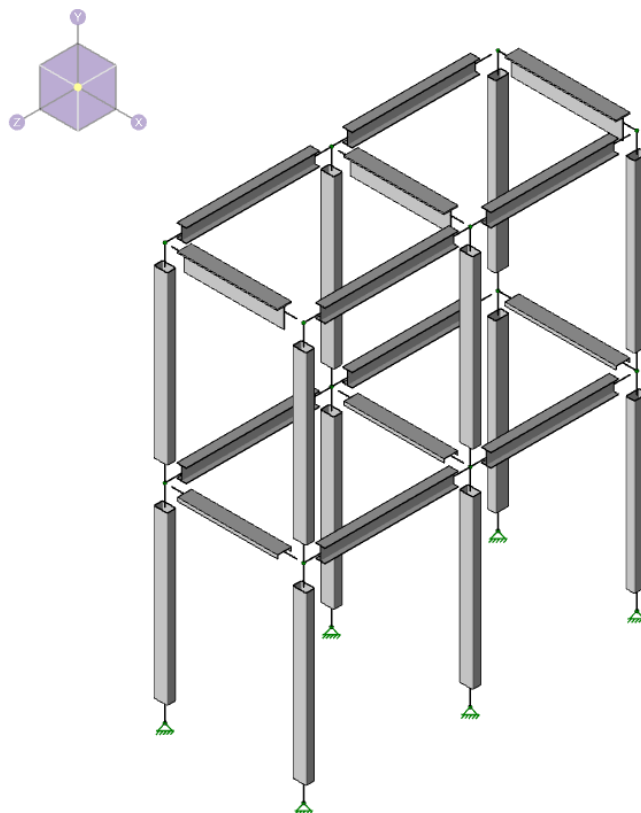


Figure 5.1 – Model Sketch

## Validation Method

Following are the hand calculations for various members for various load combinations. The steel codes used are the AISC 360-22 (16<sup>th</sup> Edition) ASD and AISC 360-22 (16<sup>th</sup> Edition) LRFD. Stiffness Reduction per the Direct Analysis Method has been turned off for this example. At least one member of each type (WF, Tee, Channel, and Tube) is validated. These hand calculation values are used to validate the results given by RISA-3D (see Tables 5.1 and 5.2).

For ASD results, set the Hot Rolled Steel code to AISC 16<sup>th</sup> (360-22): ASD and run LC 1, 2, 3, 4, 6.

For LRFD results, set the Hot Rolled Steel code to AISC 16<sup>th</sup> (360-22): LRFD and run LC 10, 11, 12, 13, 15.

## ASD Hand Calculations

### Member M10, Load Combination 1:

Shape Properties: HSS 12X8X10

$$A := 21 \cdot \text{in}^2$$

$$L := 180 \cdot \text{in}$$

$$I_y := 210 \cdot \text{in}^4$$

$$I_z := 397 \cdot \text{in}^4$$

$$Z_y := 61.9 \cdot \text{in}^3$$

$$Z_z := 82.1 \cdot \text{in}^3$$

$$h := 11.419 \cdot \text{in}$$

$$b := 6.257 \cdot \text{in}$$

$$t := 0.581 \cdot \text{in}$$

$$J := 454 \cdot \text{in}^4$$

$$S_z := 66.1 \cdot \text{in}^3$$

$$S_y := 52.5 \cdot \text{in}^3$$

Material Properties: A500 Gr. C

$$F_y := 46 \cdot \text{ksi}$$

$$E := 29000 \cdot \text{ksi}$$

$$\Omega := 1.67$$

$$K := 1.2$$

$$L_c := K \cdot L = 18 \text{ ft}$$

$$r_y := \sqrt{\frac{I_y}{A}} = 3.162 \text{ in}$$

$$r_z := \sqrt{\frac{I_z}{A}} = 4.348 \text{ in}$$

### Width to Thickness Ratios:

Compression Elements:

$$\frac{b}{t} = 10.769 < 1.4 \cdot \sqrt{\frac{E}{F_y}} = 35.152 \quad \text{Non-Slender Flange (per Table B4.1a, Case 6)}$$

$$\frac{h}{t} = 19.654 < 1.4 \cdot \sqrt{\frac{E}{F_y}} = 35.152 \quad \text{Non-Slender Web (per Table B4.1a, Case 6)}$$

Bending Elements:

$$\frac{b}{t} = 10.769 < 2.42 \cdot \sqrt{\frac{E}{F_y}} = 60.762 \quad \text{Compact Flange (per Table B4.1b, Case 19)}$$

$$\frac{h}{t} = 19.654 < 2.42 \cdot \sqrt{\frac{E}{F_y}} = 60.762 \quad \text{Compact Web (per Table B4.1b, Case 19)}$$

### Applied Loading per RISA Analysis:

Governing Location: 0 inches

$$P := 6.518 \cdot \text{kip}$$

$$M_z := 8.1909 \cdot \text{kip} \cdot \text{ft}$$

$$M_y := 1.6834 \cdot \text{kip} \cdot \text{ft}$$

Axial load at governing location

z-z Moment at governing location

y-y Moment at governing location

### Member M10, Load Combination 1, continued

---

#### Compressive Capacity:

$$F_e := \frac{(\pi^2 \cdot E)}{\left(\frac{L_c}{r_y}\right)^2} = 61.347 \text{ ksi} \quad (\text{EQN E3-4})$$

$$\frac{L_c}{r_y} = 68.305 < 4.71 \cdot \sqrt{\frac{E}{F_y}} = 118.261$$

$$\text{Therefore, } F_{cr} := (0.658)^{\left(\frac{F_y}{F_e}\right)} \cdot F_y = 33.609 \text{ ksi} \quad (\text{EQN E3-2})$$

$$P_n := F_{cr} \cdot A = 705.791 \text{ kip} \quad (\text{EQN E3-1})$$

$$P_c := \frac{P_n}{\Omega} = 422.629 \text{ kip}$$

#### Flexural Capacity:

##### Plastic Moment Yielding-

$$M_{ny\_pmy} := F_y \cdot Z_y = 237.283 \text{ kip} \cdot \text{ft} \quad (\text{EQN F7-1})$$

$$M_{nz\_pmy} := F_y \cdot Z_z = 314.717 \text{ kip} \cdot \text{ft} \quad (\text{EQN F7-1})$$

##### Flange Local Buckling-

The section is compact, so this check does not apply.

##### Web Local Buckling-

The section is compact, so this check does not apply.

##### Lateral-Torsional Buckling-

$$L_b := L = 15 \text{ ft}$$

$$L_p := 0.13 \cdot E \cdot r_y \cdot \frac{\sqrt{J \cdot A}}{M_{nz\_pmy}} = 25.686 \text{ ft} \quad (\text{EQN F7-12})$$

$$L_r := 2 \cdot E \cdot r_y \cdot \frac{\sqrt{J \cdot A}}{0.7 \cdot F_y \cdot S_z} = 701.176 \text{ ft} \quad (\text{EQN F7-13})$$

Because  $L_b < L_p$ , lateral-torsional buckling does not apply.

$$\text{Therefore, } M_{ny} := M_{ny\_pmy} = 237.283 \text{ kip} \cdot \text{ft}$$

$$M_{nz} := M_{nz\_pmy} = 314.717 \text{ kip} \cdot \text{ft}$$

$$\frac{M_{ny}}{\Omega} = 142.086 \text{ kip} \cdot \text{ft}$$

$$\frac{M_{nz}}{\Omega} = 188.453 \text{ kip} \cdot \text{ft}$$

**Member M10, Load Combination 1, continued**\_\_\_\_\_

Unity Code Check (UC Max):

$$\frac{P}{P_c} = 0.015 < 0.2$$

$$\text{Therefore, } UC_{Max} := \left( \frac{P}{2 \cdot P_c} \right) + \left( \frac{\frac{M_z}{M_{nz}}}{\Omega} \right) + \left( \frac{\frac{M_y}{M_{ny}}}{\Omega} \right) = 0.063 \quad (\text{EQN H1-1b})$$



### Member M1, Load Combination 2:

Shape Properties: HSS 12X8X10

$$A := 21 \cdot \text{in}^2$$

$$L := 180 \cdot \text{in}$$

$$I_y := 210 \cdot \text{in}^4$$

$$I_z := 397 \cdot \text{in}^4$$

$$Z_y := 61.9 \cdot \text{in}^3$$

$$Z_z := 82.1 \cdot \text{in}^3$$

$$h := 11.419 \cdot \text{in}$$

$$b := 6.257 \cdot \text{in}$$

$$t := 0.581 \cdot \text{in}$$

$$J := 454 \cdot \text{in}^4$$

$$S_z := 66.1 \cdot \text{in}^3$$

$$S_y := 52.5 \cdot \text{in}^3$$

Material Properties: A500 Gr. C

$$F_y := 46 \cdot \text{ksi}$$

$$E := 29000 \cdot \text{ksi}$$

$$\Omega := 1.67$$

$$K := 2$$

$$L_c := K \cdot L = 30 \text{ ft}$$

$$r_y := \sqrt{\frac{I_y}{A}} = 3.162 \text{ in}$$

$$r_z := \sqrt{\frac{I_z}{A}} = 4.348 \text{ in}$$

#### Width to Thickness Ratios:

Compression Elements:

$$\frac{b}{t} = 10.769 < 1.4 \cdot \sqrt{\frac{E}{F_y}} = 35.152 \quad \text{Non-Slender Flange (per Table B4.1a, Case 6)}$$

$$\frac{h}{t} = 19.654 < 1.4 \cdot \sqrt{\frac{E}{F_y}} = 35.152 \quad \text{Non-Slender Web (per Table B4.1a, Case 6)}$$

Bending Elements:

$$\frac{b}{t} = 10.769 < 2.42 \cdot \sqrt{\frac{E}{F_y}} = 60.762 \quad \text{Compact Flange (per Table B4.1b, Case 19)}$$

$$\frac{h}{t} = 19.654 < 2.42 \cdot \sqrt{\frac{E}{F_y}} = 60.762 \quad \text{Compact Web (per Table B4.1b, Case 19)}$$

#### Applied Loading per RISA Analysis:

Governing Location: 180 inches

$$P := 36.8843 \cdot \text{kip}$$

$$M_z := 32.9769 \cdot \text{kip} \cdot \text{ft}$$

$$M_y := 102.4279 \cdot \text{kip} \cdot \text{ft}$$

Axial load at governing location

z-z Moment at governing location

y-y Moment at governing location

#### Compressive Capacity:

$$F_e := \frac{(\pi^2 \cdot E)}{\left(\frac{L_c}{r_y}\right)^2} = 22.085 \text{ ksi} \quad (\text{EQN E3-4})$$

$$\frac{L_c}{r_y} = 113.842 < 4.71 \cdot \sqrt{\frac{E}{F_y}} = 118.261$$

### Member M1, Load Combination 2, continued

---

$$\text{Therefore, } F_{cr} := (0.658)^{\left(\frac{F_y}{F_e}\right)} \cdot F_y = 19.237 \text{ ksi} \quad (\text{EQN E3-2})$$

$$P_n := F_{cr} \cdot A = 403.983 \text{ kip} \quad (\text{EQN E3-1})$$

$$P_c := \frac{P_n}{\Omega} = 241.906 \text{ kip}$$

#### Flexural Capacity:

##### Plastic Moment Yielding-

$$M_{ny\_pmy} := F_y \cdot Z_y = 237.283 \text{ kip} \cdot \text{ft} \quad (\text{EQN F7-1})$$

$$M_{nz\_pmy} := F_y \cdot Z_z = 314.717 \text{ kip} \cdot \text{ft} \quad (\text{EQN F7-1})$$

##### Flange Local Buckling-

The section is compact, so this check does not apply.

##### Web Local Buckling-

The section is compact, so this check does not apply.

##### Lateral-Torsional Buckling-

$$L_b := L = 15 \text{ ft}$$

$$L_p := 0.13 \cdot E \cdot r_y \cdot \frac{\sqrt{J \cdot A}}{M_{nz\_pmy}} = 25.686 \text{ ft} \quad (\text{EQN F7-12})$$

$$L_r := 2 \cdot E \cdot r_y \cdot \frac{\sqrt{J \cdot A}}{0.7 \cdot F_y \cdot S_z} = 701.176 \text{ ft} \quad (\text{EQN F7-13})$$

Because  $L_b < L_p$ , lateral-torsional buckling does not apply.

$$\text{Therefore, } M_{ny} := M_{ny\_pmy} = 237.283 \text{ kip} \cdot \text{ft}$$

$$M_{nz} := M_{nz\_pmy} = 314.717 \text{ kip} \cdot \text{ft}$$

$$\frac{M_{ny}}{\Omega} = 142.086 \text{ kip} \cdot \text{ft}$$

$$\frac{M_{nz}}{\Omega} = 188.453 \text{ kip} \cdot \text{ft}$$

#### Unity Code Check (UC Max):

$$\frac{P}{P_c} = 0.152 < 0.2$$

$$\text{Therefore, } UC\_Max := \left( \frac{P}{2 \cdot P_c} \right) + \left( \frac{M_z}{\frac{M_{nz}}{\Omega}} \right) + \left( \frac{M_y}{\frac{M_{ny}}{\Omega}} \right) = 0.972 \quad (\text{EQN H1-1b})$$

### Member M14, Load Combination 3:

Shape Properties: C12X30

$$A := 8.81 \cdot \text{in}^2$$

$$L := 108 \cdot \text{in}$$

$$h_o := 11.5 \cdot \text{in}$$

$$I_y := 5.12 \cdot \text{in}^4$$

$$I_z := 162 \cdot \text{in}^4$$

$$Z_y := 4.32 \cdot \text{in}^3$$

$$Z_z := 33.8 \cdot \text{in}^3$$

$$S_y := 2.051 \cdot \text{in}^3$$

$$S_z := 27 \cdot \text{in}^3$$

$$C_w := 151 \cdot \text{in}^6$$

$$J := 0.861 \cdot \text{in}^4$$

$$r_{ts} := 1.01 \cdot \text{in}$$

$$b := 3.17 \cdot \text{in}$$

$$t_f := 0.501 \cdot \text{in}$$

$$t_w := 3.17 \cdot \text{in}$$

Material Properties: A36 Gr.36

$$F_y := 36 \cdot \text{ksi}$$

$$E := 29000 \cdot \text{ksi}$$

$$\Omega := 1.67$$

$$K := 1.2$$

$$r_y := \sqrt{\frac{I_y}{A}} = 0.762 \text{ in}$$

$$r_z := \sqrt{\frac{I_z}{A}} = 4.288 \text{ in}$$

### Width to Thickness Ratios:

Compression Elements:

$$\frac{b}{t_f} = 6.327 < 0.56 \cdot \sqrt{\frac{E}{F_y}} = 15.894 \quad \text{Non-Slender Flange (per Table B4.1a, Case 1)}$$

$$\frac{h}{t_w} = 3.602 < 1.49 \cdot \sqrt{\frac{E}{F_y}} = 42.29 \quad \text{Non-Slender Web (per Table B4.1a, Case 5)}$$

Bending Elements:

$$\frac{b}{t_f} = 6.327 < 0.38 \cdot \sqrt{\frac{E}{F_y}} = 10.785 \quad \text{Compact Flange (per Table B4.1b, Case 10)}$$

$$\frac{h}{t_w} = 3.602 < 3.76 \cdot \sqrt{\frac{E}{F_y}} = 106.717 \quad \text{Compact Web (per Table B4.1b, Case 15)}$$

### Applied Loading (including Torsion) per RISA Analysis:

Governing Location: 108 inches

$$P := 4.768 \cdot \text{kip}$$

$$M_{max} := 4.144 \cdot \text{kip} \cdot \text{ft}$$

$$M_A := 2.072 \cdot \text{kip} \cdot \text{ft}$$

$$M_B := 0 \cdot \text{kip} \cdot \text{ft}$$

$$M_C := 2.072 \cdot \text{kip} \cdot \text{ft}$$

Axial load at governing location

Maximum moment for Cb calculation

Moment at first quarter point for Cb calculation

Moment at halfway point for Cb calculation

Moment at third quarter point for Cb calculation

### Member M14, Load Combination 3, continued

$\sigma_{bz\_top} := 164.0898 \cdot ksi$	Local positive z bending stress at governing location
$\sigma_{by\_bot} := -1.8417 \cdot ksi$	Local positive y bending stress at governing location
$\sigma_{\omega z\_top} := -0.0295 \cdot ksi$	Local top warping bending stress (per Member Torsion spreadsheet) at governing location
$\sigma_{\omega z\_bot} := -0.0691 \cdot ksi$	Local bottom warping bending stress (per Member Torsion spreadsheet) at governing location
$Mz :=  (\sigma_{by\_bot} + \sigma_{\omega z\_bot})  \cdot Sz = 4.299 \text{ kip} \cdot ft$	z-z Moment at governing location
$My :=  (\sigma_{bz\_top} + \sigma_{\omega z\_top})  \cdot Sy = 28.041 \text{ kip} \cdot ft$	y-y Moment at governing location

### Tensile Capacity:

$$Pn := Fy \cdot A = 317.16 \text{ kip} \quad (\text{EQN D2-1})$$

$$Pt := \frac{Pn}{\Omega} = 189.916 \text{ kip}$$

### Flexural Capacity:

#### Yielding-

$$Mny := \min(Fy \cdot Zy, 1.6 \cdot Fy \cdot Sy) = 9.845 \text{ kip} \cdot ft \quad (\text{EQN F6-1})$$

$$Mnz := \min(Fy \cdot Zz, 1.6 \cdot Fy \cdot Sz) = 101.4 \text{ kip} \cdot ft \quad (\text{EQN F6-1})$$

#### Lateral Torsional Buckling-

$$c := \left(\frac{ho}{2}\right) \cdot \sqrt{\frac{Iy}{Cw}} = 1.059 \quad (\text{EQN F2-8b})$$

$$Lr := \left(\frac{1.95 \cdot rts \cdot E}{0.7 \cdot Fy}\right) \cdot \sqrt{\frac{J \cdot c}{Sz \cdot ho}} \cdot \sqrt{1 + \sqrt{1 + 6.76 \cdot \left(\frac{(0.7 \cdot Fy \cdot Sz \cdot ho)}{E \cdot J \cdot c}\right)^2}} = 15.391 \text{ ft} \quad (\text{EQN F2-6})$$

$$Lb := L = 9 \text{ ft}$$

$$Lp := 1.76 \cdot ry \cdot \sqrt{\frac{E}{Fy}} = 3.173 \text{ ft} \quad (\text{EQN F2-5})$$

$$Cb := \frac{12.5 \cdot Mmax}{2.5 \cdot Mmax + 3 \cdot MA + 4 \cdot MB + 3 \cdot MC} = 2.273 \quad (\text{EQN F1-1})$$

$$Mp := Fy \cdot Zz = 101.4 \text{ kip} \cdot ft \quad (\text{EQN F2-1})$$

$$Mnz\_ltb := \min\left(\left(Cb \cdot \left(Mp - (Mp - 0.7 \cdot Fy \cdot Sz) \cdot \left(\frac{Lb - Lp}{Lr - Lp}\right)\right)\right), Mp\right) = 101.4 \text{ kip} \cdot ft \quad (\text{EQN F2-2})$$

Therefore,  $\frac{Mny}{\Omega} = 5.895 \text{ kip} \cdot ft$

$$\frac{Mnz}{\Omega} = 60.719 \text{ kip} \cdot ft$$

**Member M14, Load Combination 3, continued**\_\_\_\_\_

Unity Code Check (UC Max):

$$\frac{P}{P_t} = 0.025 < 0.2$$

$$\text{Therefore, } UC_{Max} := \left( \frac{P}{2 \cdot P_t} \right) + \left( \frac{M_z}{\frac{M_{nz}}{\Omega}} \right) + \left( \frac{M_y}{\frac{M_{ny}}{\Omega}} \right) = 4.840 \quad (\text{EQN H1-1b})$$

### Member M25, Load Combination 2:

Shape Properties: W12x45

$$A := 13.1 \cdot \text{in}^2$$

$$L := 138 \cdot \text{in}$$

$$I_y := 50 \cdot \text{in}^4$$

$$I_z := 348 \cdot \text{in}^4$$

$$Z_y := 19 \cdot \text{in}^3$$

$$Z_z := 64.2 \cdot \text{in}^3$$

$$S_y := 12.4 \cdot \text{in}^3$$

$$S_z := 57.7 \cdot \text{in}^3$$

$$J := 1.26 \cdot \text{in}^4$$

$$r_{ts} := 2.23 \cdot \text{in}$$

$$h_o := 11.5 \cdot \text{in}$$

$$c := 1$$

$$h := 9.916 \cdot \text{in}$$

$$b := 4.025 \cdot \text{in}$$

$$t_f := 0.575 \cdot \text{in}$$

$$t_w := 0.335 \cdot \text{in}$$

Material Properties: A992

$$F_y := 50 \cdot \text{ksi}$$

$$E := 29000 \cdot \text{ksi}$$

$$\Omega := 1.67$$

$$K := 1.2$$

$$r_y := \sqrt{\frac{I_y}{A}} = 1.954 \text{ in}$$

$$r_z := \sqrt{\frac{I_z}{A}} = 5.154 \text{ in}$$

### Width to Thickness Ratios:

Compression Elements:

$$\frac{b}{t_f} = 7 < 0.56 \cdot \sqrt{\frac{E}{F_y}} = 13.487 \quad \text{Non-Slender Flange (per Table B4.1a, Case 1)}$$

$$\frac{h}{t_w} = 29.6 < 1.49 \cdot \sqrt{\frac{E}{F_y}} = 35.884 \quad \text{Non-Slender Web (per Table B4.1a, Case 5)}$$

Bending Elements:

$$\frac{b}{t_f} = 7 < 0.38 \cdot \sqrt{\frac{E}{F_y}} = 9.152 \quad \text{Compact Flange (per Table B4.1b, Case 10)}$$

$$\frac{h}{t_w} = 29.6 < 3.76 \cdot \sqrt{\frac{E}{F_y}} = 90.553 \quad \text{Compact Web (per Table B4.1b, Case 15)}$$

### Applied Loading (including Torsion) per RISA Analysis:

Governing Location: 0 inches

$$P := -0.0231 \cdot \text{kip}$$

$$M_{\max} := 7.578 \cdot \text{kip} \cdot \text{ft}$$

$$M_A := 0.018 \cdot \text{kip} \cdot \text{ft}$$

$$M_B := 3.114 \cdot \text{kip} \cdot \text{ft}$$

$$M_C := 1.707 \cdot \text{kip} \cdot \text{ft}$$

Axial load at governing location

Maximum moment for Cb calculation

Moment at first quarter point for Cb calculation

Moment at halfway point for Cb calculation

Moment at third quarter point for Cb calculation

### Member M25, Load Combination 2, continued

$$\sigma_{bz\_top} := 7.4155 \cdot ksi$$

Local positive z bending stress at governing location

$$\sigma_{by\_bot} := 1.5809 \cdot ksi$$

Local positive y bending stress at governing location

$$\sigma_{\omega z\_top} := 0.1152 \cdot ksi$$

Local top warping bending stress (per Member Torsion spreadsheet) at governing location

$$My := (\sigma_{bz\_top} + \sigma_{\omega z\_top}) Sy = 7.782 \text{ kip} \cdot ft$$

y-y Moment at governing location

$$Mz := \sigma_{by\_bot} \cdot Sz = 7.601 \text{ kip} \cdot ft$$

z-z Moment at governing location

#### Tensile Capacity:

$$Pn := Fy \cdot A = 655 \text{ kip} \quad (EQN D2-1)$$

$$Pt := \frac{Pn}{\Omega} = 392.216 \text{ kip}$$

#### Flexural Capacity:

##### **Yielding-**

$$Mny\_y := Fy \cdot Zy = 79.167 \text{ kip} \cdot ft \quad (EQN F2-1)$$

$$Mnz\_y := \min(Fy \cdot Zz, 1.6 \cdot Fy \cdot Sz) = 267.5 \text{ kip} \cdot ft \quad (EQN F6-1)$$

##### **Lateral Torsional Buckling- applies only to strong axis bending**

$$c := 1 \quad (EQN F2-8a)$$

$$Lp := 1.76 \cdot ry \cdot \sqrt{\frac{E}{Fy}} = 6.901 \text{ ft} \quad (EQN F2-5)$$

$$Lb := L = 11.5 \text{ ft}$$

$$Lr := \left( \frac{1.95 \cdot rts \cdot E}{0.7 \cdot Fy} \right) \cdot \sqrt{\left( \frac{J \cdot c}{Sz \cdot ho} \right) + \left( \sqrt{\left( \frac{J \cdot c}{Sz \cdot ho} \right)^2 + 6.76 \cdot \left( \frac{0.7 \cdot Fy}{E} \right)^2} \right)} = 22.402 \text{ ft} \quad (EQN F2-6)$$

$$Mpz := Mnz\_y = 267.5 \text{ kip} \cdot ft$$

$$Cb := \frac{12.5 \cdot Mmax}{2.5 \cdot Mmax + 3 \cdot MA + 4 \cdot MB + 3 \cdot MC} = 2.5898 \quad (EQN F1-1)$$

$$Mnz\_ltb := Cb \cdot \left( Mpz - (Mpz - 0.7 \cdot Fy \cdot Sz) \cdot \left( \frac{Lb - Lp}{Lr - Lp} \right) \right) = 616.542 \text{ kip} \cdot ft \quad (EQN F2-2)$$

##### **Flange Local Buckling- applies only to weak axis bending**

The section is compact, so this check does not apply.

Therefore,  $Mny := Mny\_y = 79.167 \text{ kip} \cdot ft$

$$Mnz := \min(Mnz\_y, Mnz\_ltb) = 267.5 \text{ kip} \cdot ft$$

$$\frac{Mny}{\Omega} = 47.405 \text{ kip} \cdot ft$$

$$\frac{Mnz}{\Omega} = 160.18 \text{ kip} \cdot ft$$

**Member M25, Load Combination 2, continued**

---

Unity Code Check (UC Max):

$$\frac{P}{P_t} = -0.0001 < 0.2$$

$$\text{Therefore, } UC_{Max} := \left( \frac{P}{2 \cdot P_t} \right) + \left( \frac{\frac{M_z}{M_{nz}}}{\Omega} \right) + \left( \frac{\frac{M_y}{M_{ny}}}{\Omega} \right) = 0.212 \quad (\text{EQN H1-1b})$$



#### Member M20, Load Combination 4:

Shape Properties: W12x45

$$A := 13.1 \cdot \text{in}^2$$

$$L := 144 \cdot \text{in}$$

$$I_y := 50 \cdot \text{in}^4$$

$$I_z := 348 \cdot \text{in}^4$$

$$Z_y := 19 \cdot \text{in}^3$$

$$Z_z := 64.2 \cdot \text{in}^3$$

$$S_y := 12.4 \cdot \text{in}^3$$

$$S_z := 57.7 \cdot \text{in}^3$$

$$J := 1.26 \cdot \text{in}^4$$

$$r_{ts} := 2.23 \cdot \text{in}$$

$$h_o := 11.5 \cdot \text{in}$$

$$c := 1$$

$$b := 4.025 \cdot \text{in}$$

$$t := 0.575 \cdot \text{in}$$

$$h := 9.916 \cdot \text{in}$$

$$b := 4.025 \cdot \text{in}$$

$$t_f := 0.575 \cdot \text{in}$$

$$t_w := 0.335 \cdot \text{in}$$

Material Properties: A992

$$F_y := 50 \cdot \text{ksi}$$

$$E := 29000 \cdot \text{ksi}$$

$$G := 11154 \cdot \text{ksi}$$

$$\Omega := 1.67$$

$$K := 1.2$$

$$r_y := \sqrt{\frac{I_y}{A}} = 1.954 \text{ in}$$

$$r_z := \sqrt{\frac{I_z}{A}} = 5.154 \text{ in}$$

#### Width to Thickness Ratios:

Compression Elements:

$$\frac{b}{t_f} = 7 < 0.56 \cdot \sqrt{\frac{E}{F_y}} = 13.487 \quad \text{Non-Slender Flange (per Table B4.1a, Case 1)}$$

$$\frac{h}{t_w} = 29.6 < 1.49 \cdot \sqrt{\frac{E}{F_y}} = 35.884 \quad \text{Non-Slender Web (per Table B4.1a, Case 5)}$$

Bending Elements:

$$\frac{b}{t_f} = 7 < 0.38 \cdot \sqrt{\frac{E}{F_y}} = 9.152 \quad \text{Compact Flange (per Table B4.1b, Case 10)}$$

$$\frac{h}{t_w} = 29.6 < 3.76 \cdot \sqrt{\frac{E}{F_y}} = 90.553 \quad \text{Compact Web (per Table B4.1b, Case 15)}$$

#### Applied Loading per RISA Analysis:

Governing Location: 144 inches

$$P := 2.304 \cdot \text{kip}$$

$$M_z := 70.831 \cdot \text{kip} \cdot \text{ft}$$

$$M_y := 0 \cdot \text{kip} \cdot \text{ft}$$

Axial load at governing location

z-z Moment at governing location

y-y Moment at governing location

## Member M20, Load Combination 4, continued

### Loading (continued):

$$M_{max} := 70.831 \cdot \text{kip} \cdot \text{ft}$$

Maximum moment for Cb calculation

$$M_A := 18.017 \cdot \text{kip} \cdot \text{ft}$$

Moment at first quarter point for Cb calculation

$$M_B := 6.698 \cdot \text{kip} \cdot \text{ft}$$

Moment at halfway point for Cb calculation

$$M_C := 36.314 \cdot \text{kip} \cdot \text{ft}$$

Moment at third quarter point for Cb calculation

### Compressive Capacity:

$$L_c := K \cdot L = 14.4 \text{ ft}$$

$$F_{e\_fb} := \frac{(\pi^2 \cdot E)}{\left(\frac{L_c}{r_y}\right)^2} = 36.585 \text{ ksi} \quad (\text{EQN E3-4})$$

$$F_{e\_fb} := \left( \frac{\pi^2 \cdot E \cdot C_w}{L_c^2} + G \cdot J \right) \cdot \left( \frac{1}{I_z + I_y} \right) = 38.948 \text{ ksi} \quad (\text{EQN E4-2})$$

$$F_e := \min(F_{e\_fb}, F_{e\_fb}) = 36.585 \text{ ksi}$$

$$\frac{L_c}{r_y} = 88.449 < 4.71 \cdot \sqrt{\frac{E}{F_y}} = 113.432$$

$$F_{cr} := (0.658)^{\left(\frac{F_y}{F_e}\right)} \cdot F_y = 28.219 \text{ ksi} \quad (\text{EQN E3-2})$$

$$P_n := F_{cr} \cdot A = 369.673 \text{ kip} \quad (\text{EQN E3-1})$$

$$P_c := \frac{P_n}{\Omega} = 221.361 \text{ kip}$$

### Flexural Capacity:

#### Yielding-

$$M_{ny\_y} := F_y \cdot Z_y = 79.167 \text{ kip} \cdot \text{ft} \quad (\text{EQN F2-1})$$

$$M_{nz\_y} := F_y \cdot Z_z = 267.5 \text{ kip} \cdot \text{ft} \quad (\text{EQN F6-1})$$

#### Lateral Torsional Buckling-

$$L_p := 1.76 \cdot r_y \cdot \sqrt{\frac{E}{F_y}} = 6.901 \text{ ft} \quad (\text{EQN F2-5})$$

$$L_b := L = 12 \text{ ft}$$

$$L_r := \left( \frac{1.95 \cdot r_{ts} \cdot E}{0.7 \cdot F_y} \right) \cdot \sqrt{\left( \frac{J \cdot c}{S_z \cdot h_o} \right)^2 + 6.76 \cdot \left( \frac{0.7 \cdot F_y}{E} \right)^2} = 22.402 \text{ ft} \quad (\text{EQN F2-6})$$

$$M_{py} := M_{ny\_y} = 79.167 \text{ kip} \cdot \text{ft}$$

$$M_{pz} := M_{nz\_y} = 267.5 \text{ kip} \cdot \text{ft}$$

**Member M20, Load Combination 4, continued**

---

$$C_b := \frac{12.5 \cdot M_{max}}{2.5 \cdot M_{max} + 3 \cdot M_A + 4 \cdot M_B + 3 \cdot M_C} = 2.413 \quad (\text{EQN F1-1})$$

$$M_{nz\_ltb} := C_b \cdot \left( M_{pz} - (M_{pz} - 0.7 \cdot F_y \cdot S_z) \cdot \left( \frac{L_b - L_p}{L_r - L_p} \right) \right) = 566.822 \text{ kip} \cdot \text{ft} \quad (\text{EQN F2-2})$$

Therefore,  $M_{ny} := M_{ny\_y} = 79.167 \text{ kip} \cdot \text{ft}$   
 $M_{nz} := \min(M_{nz\_y}, M_{nz\_ltb}) = 267.5 \text{ kip} \cdot \text{ft}$   
 $\frac{M_{ny}}{\Omega} = 47.405 \text{ kip} \cdot \text{ft}$   
 $\frac{M_{nz}}{\Omega} = 160.18 \text{ kip} \cdot \text{ft}$

**Unity Code Check (UC Max):**

$$\frac{P}{P_c} = 0.01 < 0.2$$

Therefore,  $UC\_Max := \left( \frac{P}{2 \cdot P_c} \right) + \left( \frac{M_z}{\frac{M_{nz}}{\Omega}} \right) + \left( \frac{M_y}{\frac{M_{ny}}{\Omega}} \right) = 0.447 \quad (\text{EQN H1-1b})$

### Member M16, Load Combination 6:

Shape Properties: WT18x85

$$A := 25 \cdot \text{in}^2$$

$$L := 120 \cdot \text{in}$$

$$I_y := 160 \cdot \text{in}^4$$

$$I_z := 786 \cdot \text{in}^4$$

$$Z_y := 41.8 \cdot \text{in}^3$$

$$Z_z := 105 \cdot \text{in}^3$$

$$S_y := 26.6 \cdot \text{in}^3$$

$$S_z := 58.9 \cdot \text{in}^3$$

$$J := 7.51 \cdot \text{in}^4$$

$$C_w := 63.2 \cdot \text{in}^6$$

$$r_o := 7.437 \cdot \text{in}$$

$$y_{\text{bar}} := 4.73 \cdot \text{in}$$

$$x_o := 0 \cdot \text{in}$$

$$y_o := 4.18 \cdot \text{in}$$

$$d := 18.1 \cdot \text{in}$$

$$b := 6 \cdot \text{in}$$

$$t_f := 1.1 \cdot \text{in}$$

$$t_w := 0.68 \cdot \text{in}$$

Material Properties: A36 Gr.36

$$F_y := 36 \cdot \text{ksi}$$

$$E := 29000 \cdot \text{ksi}$$

$$G := 11154 \cdot \text{ksi}$$

$$\Omega := 1.67$$

$$K := 1.2$$

$$r_y := \sqrt{\frac{I_y}{A}} = 2.53 \text{ in}$$

$$r_z := \sqrt{\frac{I_z}{A}} = 5.607 \text{ in}$$

### Width to Thickness Ratios:

Compression Elements:

$$\frac{b}{t_f} = 5.455 < 0.56 \cdot \sqrt{\frac{E}{F_y}} = 15.894 \quad \text{Non-Slender Flange (per Table B4.1a, Case 1)}$$

$$\frac{d}{t_w} = 26.618 > 0.75 \cdot \sqrt{\frac{E}{F_y}} = 21.287 \quad \text{Slender Web (per Table B4.1a, Case 4)}$$

Bending Elements:

$$\frac{b}{t_f} = 5.455 < 0.38 \cdot \sqrt{\frac{E}{F_y}} = 10.785 \quad \text{Compact Flange (per Table B4.1b, Case 10)}$$

$$\frac{d}{t_w} = 26.618 < 1.52 \cdot \sqrt{\frac{E}{F_y}} = 43.141 \quad \text{Non-Compact Web (per Table B4.1b, Case 14)}$$

### Applied Loading (including Torsion) per RISA Analysis:

Governing Location: 0 inches

$$P := 3.3436 \cdot \text{kip}$$

$$\sigma_{bz\_top} := 1.0566 \cdot \text{ksi}$$

$$\sigma_{by\_bot} := 21.297 \cdot \text{ksi}$$

$$\sigma_{\omega z\_top} := 0 \cdot \text{ksi}$$

Axial load at governing location

Local positive z bending stress at governing location

Local positive y bending stress at governing location  
Local top warping bending stress (per Member

Torsion spreadsheet) at governing location

### Member M16, Load Combination 6, continued

$$M_y := (\sigma_{bz\_top} + \sigma_{\omega z\_top}) S_y = 2.342 \text{ kip} \cdot \text{ft}$$

y-y Moment at governing location

$$M_z := \sigma_{by\_bot} \cdot S_z = 104.533 \text{ kip} \cdot \text{ft}$$

z-z Moment at governing location

#### Compressive Capacity:

$$\lambda := \frac{d}{tw} = 26.618$$

Slender compression web width to thickness ratio per Table B4.1a (case 4)

$$\lambda_r := 0.75 \cdot \sqrt{\frac{E}{F_y}} = 21.287$$

Limiting width to thickness ratio per Table B4.1a (case 4)

$$L_c := K \cdot L = 12 \text{ ft}$$

$$F_{eE3} := \frac{\pi^2 \cdot E}{\left(\frac{L_c}{r_y}\right)^2} = 88.339 \text{ ksi}$$

(EQN E3-4)

$$F_{ey} := \frac{\pi^2 \cdot E}{\left(\frac{L_c}{r_y}\right)^2} = 88.339 \text{ ksi}$$

(EQN E4-6)

$$F_{ez} := \left( \frac{\pi^2 \cdot E \cdot \text{in}^6}{L_c^2} + G \cdot J \right) \cdot \frac{1}{A \cdot r_o^2} = 60.591 \text{ ksi}$$

(EQN E4-7) Note C<sub>w</sub> is omitted for WT per User Note on page 16.1-37

$$H := 1 - \frac{x_o^2 + y_o^2}{r_o^2} = 0.684$$

(EQN E4-8)

$$F_{eE4} := \left( \frac{F_{ey} + F_{ez}}{2 \cdot H} \right) \cdot \left( 1 - \sqrt{1 - \frac{4 \cdot F_{ey} \cdot F_{ez} \cdot H}{(F_{ey} + F_{ez})^2}} \right) = 45.413 \text{ ksi}$$

(EQN E4-3)

$$F_e := \min(F_{eE3}, F_{eE4}) = 45.413 \text{ ksi}$$

$$\frac{F_y}{F_e} = 0.793 < 2.25$$

$$F_n := \left( 0.658^{\frac{F_y}{F_e}} \right) \cdot F_y = 25.835 \text{ ksi}$$

(EQN E3-2)

$$\lambda = 26.618 > \lambda_r \cdot \sqrt{\frac{F_y}{F_n}} = 25.128$$

$$c1 := 0.22$$

Effective width imperfection adjustment factors per Table E7.1

$$c2 := \frac{1 - \sqrt{1 - 4 \cdot c1}}{2 \cdot c1} = 1.485$$

(EQN E7-4)

$$F_{el} := \left( c2 \cdot \frac{\lambda_r}{\lambda} \right)^2 \cdot F_y = 50.803 \text{ ksi}$$

(EQN E7-5)

$$d_e := d \cdot \left( 1 - c1 \cdot \sqrt{\frac{F_{el}}{F_n}} \right) \cdot \sqrt{\frac{F_{el}}{F_n}} = 17.551 \text{ in}$$

(EQN E7-3)

### Member M16, Load Combination 6, continued

$$Ae := A - ((d - de) \cdot tw) = 24.627 \text{ in}^2$$

Summation of effective areas based on the reduced effective width, be

$$Pn := Fcr \cdot Ae = 636.231 \text{ kip}$$

(EQN E7-1)

$$Pnc := \frac{Pn}{\Omega} = 380.977 \text{ kip}$$

#### Flexural Capacity:

##### Yielding-

$$Mny_y := \min(Fy \cdot Zy, 1.6 \cdot Fy \cdot Sy) = 125.4 \text{ kip} \cdot \text{ft} \quad (\text{EQN F9-4})$$

$$Mnz_y := \min(Fy \cdot Zz, 1.6 \cdot Fy \cdot Sz) = 282.72 \text{ kip} \cdot \text{ft} \quad (\text{EQN F9-4})$$

##### Lateral Torsional Buckling-

$$B := -2.3 \cdot \left(\frac{d}{L}\right) \cdot \sqrt{\frac{Iy}{J}} = -1.601 \quad (\text{EQN F9-12})$$

$$Mcr := \frac{1.95 \cdot E}{L} \cdot \sqrt{Iy \cdot J} \cdot (B + \sqrt{1 + B^2}) = 390.149 \text{ kip} \cdot \text{ft} \quad (\text{EQN F9-13})$$

$$Mnz_{ltb} := \min(Mcr, Fy \cdot Sz) = 176.7 \text{ kip} \cdot \text{ft} \quad (\text{EQN F9-4})$$

##### Flange Local Buckling-

The flange is compact and in compression, so this check does not apply.

##### Local Buckling of Tee Stems in Flexural Compression-

$$0.84 \cdot \sqrt{\frac{E}{Fy}} = 23.841 < \frac{d}{tw} = 26.618 < 1.52 \cdot \sqrt{\frac{E}{Fy}} = 43.141$$

$$Fcr_b := \left(1.43 - 0.515 \cdot \left(\frac{d}{tw}\right) \cdot \sqrt{\frac{Fy}{E}}\right) \cdot Fy = 34.093 \text{ ksi} \quad (\text{EQN F9-18})$$

$$Mnz_{lb} := Fcr_b \cdot Sz = 167.338 \text{ kip} \cdot \text{ft} \quad (\text{EQN F9-16})$$

Therefore,  $Mny := Mny_y = 125.4 \text{ kip} \cdot \text{ft}$

$$Mnz := \min(Mnz_y, Mnz_{ltb}, Mnz_{lb}) = 167.338 \text{ kip} \cdot \text{ft}$$

$$\frac{Mny}{\Omega} = 75.09 \text{ kip} \cdot \text{ft}$$

$$\frac{Mnz}{\Omega} = 100.203 \text{ kip} \cdot \text{ft}$$

#### Unity Code Check (UC Max):

$$\frac{P}{Pnc} = 0.009 < 0.2$$

$$\text{Therefore, } UC_{Max} := \left(\frac{P}{2 \cdot Pnc}\right) + \left(\frac{Mz}{\frac{Mnz}{\Omega}}\right) + \left(\frac{My}{\frac{Mny}{\Omega}}\right) = 1.079 \quad (\text{EQN H1-1b})$$

## ASD Results Comparison

ASD Unity Check Comparisons				
Member	Load Combination	RISA-3D	Hand Calculations	% Difference
M10	1	0.063	0.063	0.00
M1	2	0.972	0.972	0.00
M14	3	4.840	4.840	0.00
M25	2	0.212	0.212	0.00
M20	4	0.447	0.447	0.00
M16	6	1.079	1.079	0.00

Table 5.1 – ASD Comparisons

## Conclusion

As can be seen in the chart above, the results match exactly.

## LRFD Hand Calculations

### Member M10, Load Combination 10:

Shape Properties: HSS 12X8X10

$$A := 21 \cdot \text{in}^2$$

$$L := 180 \cdot \text{in}$$

$$I_y := 210 \cdot \text{in}^4$$

$$I_z := 397 \cdot \text{in}^4$$

$$Z_y := 61.9 \cdot \text{in}^3$$

$$Z_z := 82.1 \cdot \text{in}^3$$

$$h := 11.419 \cdot \text{in}$$

$$b := 6.257 \cdot \text{in}$$

$$t := 0.581 \cdot \text{in}$$

$$J := 454 \cdot \text{in}^4$$

$$S_z := 66.1 \cdot \text{in}^3$$

$$S_y := 52.5 \cdot \text{in}^3$$

Material Properties: A500 Gr. C

$$F_y := 46 \cdot \text{ksi}$$

$$E := 29000 \cdot \text{ksi}$$

$$\phi := 0.9$$

$$K := 1.2$$

$$L_c := K \cdot L = 18 \text{ ft}$$

$$r_y := \sqrt{\frac{I_y}{A}} = 3.162 \text{ in}$$

$$r_z := \sqrt{\frac{I_z}{A}} = 4.348 \text{ in}$$

### Width to Thickness Ratios:

Compression Elements:

$$\frac{b}{t} = 10.769 < 1.4 \cdot \sqrt{\frac{E}{F_y}} = 35.152 \quad \text{Non-Slender Flange (per Table B4.1a, Case 6)}$$

$$\frac{h}{t} = 19.654 < 1.4 \cdot \sqrt{\frac{E}{F_y}} = 35.152 \quad \text{Non-Slender Web (per Table B4.1a, Case 6)}$$

Bending Elements:

$$\frac{b}{t} = 10.769 < 2.42 \cdot \sqrt{\frac{E}{F_y}} = 60.762 \quad \text{Compact Flange (per Table B4.1b, Case 19)}$$

$$\frac{h}{t} = 19.654 < 2.42 \cdot \sqrt{\frac{E}{F_y}} = 60.762 \quad \text{Compact Web (per Table B4.1b, Case 19)}$$

### Applied Loading per RISA Analysis:

Governing Location: 0 inches

$$P := 8.154 \cdot \text{kip}$$

$$M_z := 11.791 \cdot \text{kip} \cdot \text{ft}$$

$$M_y := 2.02 \cdot \text{kip} \cdot \text{ft}$$

Axial load at governing location

z-z Moment at governing location

y-y Moment at governing location



### Member M10, Load Combination 10, continued

#### Compressive Capacity:

$$F_e := \frac{(\pi^2 \cdot E)}{\left(\frac{L_c}{r_y}\right)^2} = 61.347 \text{ ksi} \quad (\text{EQN E3-4})$$

$$\frac{L_c}{r_y} = 68.305 < 4.71 \cdot \sqrt{\frac{E}{F_y}} = 118.261$$

$$\text{Therefore, } F_{cr} := \left(0.658\right)^{\left(\frac{F_y}{F_e}\right)} \cdot F_y = 33.609 \text{ ksi} \quad (\text{EQN E3-2})$$

$$P_n := F_{cr} \cdot A = 705.791 \text{ kip} \quad (\text{EQN E3-1})$$

$$P_c := \phi \cdot P_n = 635.212 \text{ kip}$$

#### Flexural Capacity:

##### Plastic Moment Yielding-

$$M_{ny\_pmy} := F_y \cdot Z_y = 237.283 \text{ kip} \cdot \text{ft} \quad (\text{EQN F7-1})$$

$$M_{nz\_pmy} := F_y \cdot Z_z = 314.717 \text{ kip} \cdot \text{ft} \quad (\text{EQN F7-1})$$

##### Flange Local Buckling-

The section is compact, so this check does not apply.

##### Web Local Buckling-

The section is compact, so this check does not apply.

##### Lateral-Torsional Buckling-

$$L_b := L = 15 \text{ ft}$$

$$L_p := 0.13 \cdot E \cdot r_y \cdot \frac{\sqrt{J \cdot A}}{M_{nz\_pmy}} = 25.686 \text{ ft} \quad (\text{EQN F7-12})$$

$$L_r := 2 \cdot E \cdot r_y \cdot \frac{\sqrt{J \cdot A}}{0.7 \cdot F_y \cdot S_z} = 701.176 \text{ ft} \quad (\text{EQN F7-13})$$

Because  $L_b < L_p$ , lateral-torsional buckling does not apply.

$$\text{Therefore, } M_{ny} := M_{ny\_pmy} = 237.283 \text{ kip} \cdot \text{ft}$$

$$M_{nz} := M_{nz\_pmy} = 314.717 \text{ kip} \cdot \text{ft}$$

$$\phi \cdot M_{ny} = 213.555 \text{ kip} \cdot \text{ft}$$

$$\phi \cdot M_{nz} = 283.245 \text{ kip} \cdot \text{ft}$$

#### Unity Code Check (UC Max):

$$\frac{P}{P_c} = 0.013 < 0.2$$

$$\text{Therefore, } UC\_Max := \left(\frac{P}{2 \cdot P_c}\right) + \left(\frac{M_z}{\phi \cdot M_{nz}}\right) + \left(\frac{M_y}{\phi \cdot M_{ny}}\right) = 0.058 \quad (\text{EQN H1-1b})$$

### Member M1, Load Combination 11:

Shape Properties: HSS 12X8X10

$$A := 21 \cdot \text{in}^2$$

$$L := 180 \cdot \text{in}$$

$$I_y := 210 \cdot \text{in}^4$$

$$I_z := 397 \cdot \text{in}^4$$

$$Z_y := 61.9 \cdot \text{in}^3$$

$$Z_z := 82.1 \cdot \text{in}^3$$

$$h := 11.419 \cdot \text{in}$$

$$b := 6.257 \cdot \text{in}$$

$$t := 0.581 \cdot \text{in}$$

$$J := 454 \cdot \text{in}^4$$

$$S_z := 66.1 \cdot \text{in}^3$$

$$S_y := 52.5 \cdot \text{in}^3$$

Material Properties: A500 Gr. C

$$F_y := 46 \cdot \text{ksi}$$

$$E := 29000 \cdot \text{ksi}$$

$$\phi := 0.9$$

$$K := 2$$

$$L_c := K \cdot L = 30 \text{ ft}$$

$$r_y := \sqrt{\frac{I_y}{A}} = 3.162 \text{ in}$$

$$r_z := \sqrt{\frac{I_z}{A}} = 4.348 \text{ in}$$

### Width to Thickness Ratios:

Compression Elements:

$$\frac{b}{t} = 10.769 < 1.4 \cdot \sqrt{\frac{E}{F_y}} = 35.152 \quad \text{Non-Slender Flange (per Table B4.1a, Case 6)}$$

$$\frac{h}{t} = 19.654 < 1.4 \cdot \sqrt{\frac{E}{F_y}} = 35.152 \quad \text{Non-Slender Web (per Table B4.1a, Case 6)}$$

Bending Elements:

$$\frac{b}{t} = 10.769 < 2.42 \cdot \sqrt{\frac{E}{F_y}} = 60.762 \quad \text{Compact Flange (per Table B4.1b, Case 19)}$$

$$\frac{h}{t} = 19.654 < 2.42 \cdot \sqrt{\frac{E}{F_y}} = 60.762 \quad \text{Compact Web (per Table B4.1b, Case 19)}$$

### Applied Loading per RISA Analysis:

Governing Location: 180 inches

$$P := 42.792 \cdot \text{kip}$$

$$M_z := 39.005 \cdot \text{kip} \cdot \text{ft}$$

$$M_y := 125.186 \cdot \text{kip} \cdot \text{ft}$$

Axial load at governing location

z-z Moment at governing location

y-y Moment at governing location

### Compressive Capacity:

$$F_e := \frac{(\pi^2 \cdot E)}{\left(\frac{L_c}{r_y}\right)^2} = 22.085 \text{ ksi} \quad (\text{EQN E3-4})$$

$$\frac{L_c}{r_y} = 113.842 < 4.71 \cdot \sqrt{\frac{E}{F_y}} = 118.261$$

### Member M1, Load Combination 11, continued

---

$$\text{Therefore, } F_{cr} := \left(0.658\right)^{\left(\frac{F_y}{F_e}\right)} \cdot F_y = 19.237 \text{ ksi} \quad (\text{EQN E3-2})$$

$$P_n := F_{cr} \cdot A = 403.983 \text{ kip} \quad (\text{EQN E3-1})$$

$$P_c := \phi \cdot P_n = 363.584 \text{ kip}$$

#### Flexural Capacity:

##### Plastic Moment Yielding-

$$M_{ny\_pmy} := F_y \cdot Z_y = 237.283 \text{ kip} \cdot \text{ft} \quad (\text{EQN F7-1})$$

$$M_{nz\_pmy} := F_y \cdot Z_z = 314.717 \text{ kip} \cdot \text{ft} \quad (\text{EQN F7-1})$$

##### Flange Local Buckling-

The section is compact, so this check does not apply.

##### Web Local Buckling-

The section is compact, so this check does not apply.

##### Lateral-Torsional Buckling-

$$L_b := L = 15 \text{ ft}$$

$$L_p := 0.13 \cdot E \cdot r_y \cdot \frac{\sqrt{J \cdot A}}{M_{nz\_pmy}} = 25.686 \text{ ft} \quad (\text{EQN F7-12})$$

$$L_r := 2 \cdot E \cdot r_y \cdot \frac{\sqrt{J \cdot A}}{0.7 \cdot F_y \cdot S_z} = 701.176 \text{ ft} \quad (\text{EQN F7-13})$$

Because  $L_b < L_p$ , lateral-torsional buckling does not apply.

$$\text{Therefore, } M_{ny} := M_{ny\_pmy} = 237.283 \text{ kip} \cdot \text{ft}$$

$$M_{nz} := M_{nz\_pmy} = 314.717 \text{ kip} \cdot \text{ft}$$

$$\phi \cdot M_{ny} = 213.555 \text{ kip} \cdot \text{ft}$$

$$\phi \cdot M_{nz} = 283.245 \text{ kip} \cdot \text{ft}$$

#### Unity Code Check (UC Max):

$$\frac{P}{P_c} = 0.118 < 0.2$$

$$\text{Therefore, } UC\_Max := \left(\frac{P}{2 \cdot P_c}\right) + \left(\frac{M_z}{(\phi \cdot M_{nz})}\right) + \left(\frac{M_y}{(\phi \cdot M_{ny})}\right) = 0.783 \quad (\text{EQN H1-1b})$$

### Member M14, Load Combination 12:

Shape Properties: C12X30

$$A := 8.81 \cdot \text{in}^2$$

$$L := 108 \cdot \text{in}$$

$$h_o := 11.5 \cdot \text{in}$$

$$I_y := 5.12 \cdot \text{in}^4$$

$$I_z := 162 \cdot \text{in}^4$$

$$Z_y := 4.32 \cdot \text{in}^3$$

$$Z_z := 33.8 \cdot \text{in}^3$$

$$S_y := 2.051 \cdot \text{in}^3$$

$$S_z := 27 \cdot \text{in}^3$$

$$C_w := 151 \cdot \text{in}^6$$

$$J := 0.861 \cdot \text{in}^4$$

$$r_{ts} := 1.01 \cdot \text{in}$$

$$b := 3.17 \cdot \text{in}$$

$$t_f := 0.501 \cdot \text{in}$$

$$t_w := 3.17 \cdot \text{in}$$

Material Properties: A36 Gr.36

$$F_y := 36 \cdot \text{ksi}$$

$$E := 29000 \cdot \text{ksi}$$

$$\phi := 0.9$$

$$K := 1.2$$

$$r_y := \sqrt{\frac{I_y}{A}} = 0.762 \text{ in}$$

$$r_z := \sqrt{\frac{I_z}{A}} = 4.288 \text{ in}$$

### Width to Thickness Ratios:

Compression Elements:

$$\frac{b}{t_f} = 6.327 < 0.56 \cdot \sqrt{\frac{E}{F_y}} = 15.894 \quad \text{Non-Slender Flange (per Table B4.1a, Case 1)}$$

$$\frac{h}{t_w} = 3.602 < 1.49 \cdot \sqrt{\frac{E}{F_y}} = 42.29 \quad \text{Non-Slender Web (per Table B4.1a, Case 5)}$$

Bending Elements:

$$\frac{b}{t_f} = 6.327 < 0.38 \cdot \sqrt{\frac{E}{F_y}} = 10.785 \quad \text{Compact Flange (per Table B4.1b, Case 10)}$$

$$\frac{h}{t_w} = 3.602 < 3.76 \cdot \sqrt{\frac{E}{F_y}} = 106.717 \quad \text{Compact Web (per Table B4.1b, Case 15)}$$

### Applied Loading (including Torsion) per RISA Analysis:

Governing Location: 108 inches

$$P := 5.425 \cdot \text{kip}$$

$$M_{max} := 5.055 \cdot \text{kip} \cdot \text{ft}$$

$$M_A := 2.527 \cdot \text{kip} \cdot \text{ft}$$

$$M_B := 0 \cdot \text{kip} \cdot \text{ft}$$

$$M_C := 2.528 \cdot \text{kip} \cdot \text{ft}$$

Axial load at governing location

Maximum moment for Cb calculation

Moment at first quarter point for Cb calculation

Moment at halfway point for Cb calculation

Moment at third quarter point for Cb calculation

### Member M14, Load Combination 12, continued

$\sigma_{bz\_top} := 199.4163 \cdot ksi$	Local positive z bending stress at governing location
$\sigma_{by\_bot} := -2.2467 \cdot ksi$	Local positive y bending stress at governing location
$\sigma_{\omega z\_top} := -0.0363 \cdot ksi$	Local top warping bending stress (per Member Torsion spreadsheet) at governing location
$\sigma_{\omega z\_bot} := -0.0848 \cdot ksi$	Local bottom warping bending stress (per Member Torsion spreadsheet) at governing location
$Mz := \left( \sigma_{by\_bot} + \sigma_{\omega z\_bot} \right) \cdot Sz = 5.246 \text{ kip} \cdot ft$	z-z Moment at governing location
$My := \left( \sigma_{bz\_top} + \sigma_{\omega z\_top} \right) \cdot Sy = 34.077 \text{ kip} \cdot ft$	y-y Moment at governing location

#### Tensile Capacity:

$$Pn := Fy \cdot A = 317.16 \text{ kip} \quad (EQN D2-1)$$

$$Pt := \phi \cdot Pn = 285.444 \text{ kip}$$

#### Flexural Capacity:

##### Yielding-

$$Mny := \min(Fy \cdot Zy, 1.6 \cdot Fy \cdot Sy) = 9.845 \text{ kip} \cdot ft \quad (EQN F6-1)$$

$$Mnz := \min(Fy \cdot Zz, 1.6 \cdot Fy \cdot Sz) = 101.4 \text{ kip} \cdot ft \quad (EQN F6-1)$$

##### Lateral Torsional Buckling-

$$c := \left( \frac{ho}{2} \right) \cdot \sqrt{\frac{Iy}{Cw}} = 1.059 \quad (EQN F2-8b)$$

$$Lr := \left( \frac{1.95 \cdot rts \cdot E}{0.7 \cdot Fy} \right) \cdot \sqrt{\frac{J \cdot c}{Sz \cdot ho}} \cdot \sqrt{1 + \sqrt{1 + 6.76 \cdot \left( \frac{(0.7 \cdot Fy \cdot Sz \cdot ho)^2}{E \cdot J \cdot c} \right)}} = 15.391 \text{ ft} \quad (EQN F2-6)$$

$$Lb := L = 9 \text{ ft}$$

$$Lp := 1.76 \cdot ry \cdot \sqrt{\frac{E}{Fy}} = 3.173 \text{ ft} \quad (EQN F2-5)$$

$$Cb := \frac{12.5 \cdot Mmax}{2.5 \cdot Mmax + 3 \cdot MA + 4 \cdot MB + 3 \cdot MC} = 2.273 \quad (EQN F1-1)$$

$$Mp := Fy \cdot Zz = 101.4 \text{ kip} \cdot ft \quad (EQN F2-1)$$

$$Mnz\_ltb := \min \left( \left( Cb \cdot \left( Mp - (Mp - 0.7 \cdot Fy \cdot Sz) \cdot \left( \frac{Lb - Lp}{Lr - Lp} \right) \right) \right), Mp \right) = 101.4 \text{ kip} \cdot ft \quad (EQN F2-2)$$

Therefore,

$$\phi \cdot Mny = 8.86 \text{ kip} \cdot ft$$

$$\phi \cdot Mnz = 91.26 \text{ kip} \cdot ft$$

**Member M14, Load Combination 12, continued**\_\_\_\_\_

Unity Code Check (UC Max):

$$\frac{P}{P_t} = 0.019 < 0.2$$

$$\text{Therefore, } UC_{Max} := \left( \frac{P}{2 \cdot P_t} \right) + \left( \frac{M_z}{(\phi \cdot M_{nz})} \right) + \left( \frac{M_y}{(\phi \cdot M_{ny})} \right) = 3.913 \quad (\text{EQN H1-1b})$$

### Member M25, Load Combination 11:

Shape Properties: W12x45

$$A := 13.1 \cdot \text{in}^2$$

$$L := 138 \cdot \text{in}$$

$$I_y := 50 \cdot \text{in}^4$$

$$I_z := 348 \cdot \text{in}^4$$

$$Z_y := 19 \cdot \text{in}^3$$

$$Z_z := 64.2 \cdot \text{in}^3$$

$$S_y := 12.4 \cdot \text{in}^3$$

$$S_z := 57.7 \cdot \text{in}^3$$

$$J := 1.26 \cdot \text{in}^4$$

$$r_{ts} := 2.23 \cdot \text{in}$$

$$h_o := 11.5 \cdot \text{in}$$

$$c := 1$$

$$h := 9.916 \cdot \text{in}$$

$$b := 4.025 \cdot \text{in}$$

$$t_f := 0.575 \cdot \text{in}$$

$$t_w := 0.335 \cdot \text{in}$$

Material Properties: A992

$$F_y := 50 \cdot \text{ksi}$$

$$E := 29000 \cdot \text{ksi}$$

$$\phi := 0.9$$

$$K := 1.2$$

$$r_y := \sqrt{\frac{I_y}{A}} = 1.954 \text{ in}$$

$$r_z := \sqrt{\frac{I_z}{A}} = 5.154 \text{ in}$$

### Width to Thickness Ratios:

Compression Elements:

$$\frac{b}{t_f} = 7 < 0.56 \cdot \sqrt{\frac{E}{F_y}} = 13.487 \quad \text{Non-Slender Flange (per Table B4.1a, Case 1)}$$

$$\frac{h}{t_w} = 29.6 < 1.49 \cdot \sqrt{\frac{E}{F_y}} = 35.884 \quad \text{Non-Slender Web (per Table B4.1a, Case 5)}$$

Bending Elements:

$$\frac{b}{t_f} = 7 < 0.38 \cdot \sqrt{\frac{E}{F_y}} = 9.152 \quad \text{Compact Flange (per Table B4.1b, Case 10)}$$

$$\frac{h}{t_w} = 29.6 < 3.76 \cdot \sqrt{\frac{E}{F_y}} = 90.553 \quad \text{Compact Web (per Table B4.1b, Case 15)}$$

### Applied Loading (including Torsion) per RISA Analysis:

Governing Location: 0 inches

$$P := 0.249 \cdot \text{kip}$$

$$M_{max} := 8.552 \cdot \text{kip} \cdot \text{ft}$$

$$M_A := 0.998 \cdot \text{kip} \cdot \text{ft}$$

$$M_B := 2.504 \cdot \text{kip} \cdot \text{ft}$$

$$M_C := 1.956 \cdot \text{kip} \cdot \text{ft}$$

Axial load at governing location

Maximum moment for Cb calculation

Moment at first quarter point for Cb calculation

Moment at halfway point for Cb calculation

Moment at third quarter point for Cb calculation

### Member M25, Load Combination 11, continued

$\sigma_{bz\_top} := 9.3937 \cdot ksi$	Local positive z bending stress at governing location
$\sigma_{by\_bot} := 1.7842 \cdot ksi$	Local positive y bending stress at governing location
$\sigma_{\omega z\_top} := 0.1453 \cdot ksi$	Local top warping bending stress (per Member Torsion spreadsheet) at governing location
$M_y := (\sigma_{bz\_top} + \sigma_{\omega z\_top}) \cdot S_y = 9.857 \text{ kip} \cdot ft$	y-y Moment at governing location
$M_z := \sigma_{by\_bot} \cdot S_z = 8.579 \text{ kip} \cdot ft$	z-z Moment at governing location

#### Tensile Capacity:

$$P_n := F_y \cdot A = 655 \text{ kip} \quad (\text{EQN D2-1})$$

$$P_t := \phi \cdot P_n = 589.5 \text{ kip}$$

#### Flexural Capacity:

##### **Yielding-**

$$M_{ny\_y} := F_y \cdot Z_y = 79.167 \text{ kip} \cdot ft \quad (\text{EQN F2-1})$$

$$M_{nz\_y} := \min(F_y \cdot Z_z, 1.6 \cdot F_y \cdot S_z) = 267.5 \text{ kip} \cdot ft \quad (\text{EQN F6-1})$$

##### **Lateral Torsional Buckling- applies only to strong axis bending**

$$c := 1 \quad (\text{EQN F2-8a})$$

$$L_p := 1.76 \cdot r_y \cdot \sqrt{\frac{E}{F_y}} = 6.901 \text{ ft} \quad (\text{EQN F2-5})$$

$$L_b := L = 11.5 \text{ ft}$$

$$L_r := \left( \frac{1.95 \cdot r_{ts} \cdot E}{0.7 \cdot F_y} \right) \cdot \sqrt{\left( \frac{J \cdot c}{S_z \cdot h_o} \right) + \left( \sqrt{\left( \frac{J \cdot c}{S_z \cdot h_o} \right)^2 + 6.76 \cdot \left( \frac{0.7 \cdot F_y}{E} \right)^2} \right)} = 22.402 \text{ ft} \quad (\text{EQN F2-6})$$

$$M_{pz} := M_{nz\_y} = 267.5 \text{ kip} \cdot ft$$

$$C_b := \frac{12.5 \cdot M_{max}}{2.5 \cdot M_{max} + 3 \cdot M_A + 4 \cdot M_B + 3 \cdot M_C} = 2.6554 \quad (\text{EQN F1-1})$$

$$M_{nz\_ltb} := C_b \cdot \left( M_{pz} - (M_{pz} - 0.7 \cdot F_y \cdot S_z) \cdot \left( \frac{L_b - L_p}{L_r - L_p} \right) \right) = 632.15 \text{ kip} \cdot ft \quad (\text{EQN F2-2})$$

##### **Flange Local Buckling- applies only to weak axis bending**

The section is compact, so this check does not apply.

Therefore,  $M_{ny} := M_{ny\_y} = 79.167 \text{ kip} \cdot ft$

$$M_{nz} := \min(M_{nz\_y}, M_{nz\_ltb}) = 267.5 \text{ kip} \cdot ft$$

$$\phi \cdot M_{ny} = 71.25 \text{ kip} \cdot ft$$

$$\phi \cdot M_{nz} = 240.75 \text{ kip} \cdot ft$$



**Member M25, Load Combination 11, continued**\_\_\_\_\_

Unity Code Check (UC Max):

$$\frac{P}{P_t} = 0.0004 < 0.2$$

$$\text{Therefore, } UC_{Max} := \left( \frac{P}{2 \cdot P_t} \right) + \left( \frac{M_z}{\phi \cdot M_{nz}} \right) + \left( \frac{M_y}{\phi \cdot M_{ny}} \right) = 0.174 \quad (\text{EQN H1-1b})$$

### Member M20, Load Combination 13:

Shape Properties: W12x45

$$A := 13.1 \cdot \text{in}^2$$

$$L := 144 \cdot \text{in}$$

$$I_y := 50 \cdot \text{in}^4$$

$$I_z := 348 \cdot \text{in}^4$$

$$Z_y := 19 \cdot \text{in}^3$$

$$Z_z := 64.2 \cdot \text{in}^3$$

$$S_y := 12.4 \cdot \text{in}^3$$

$$S_z := 57.7 \cdot \text{in}^3$$

$$J := 1.26 \cdot \text{in}^4$$

$$r_{ts} := 2.23 \cdot \text{in}$$

$$h_o := 11.5 \cdot \text{in}$$

$$c := 1$$

$$b := 4.025 \cdot \text{in}$$

$$t := 0.575 \cdot \text{in}$$

$$h := 9.916 \cdot \text{in}$$

$$b := 4.025 \cdot \text{in}$$

$$t_f := 0.575 \cdot \text{in}$$

$$t_w := 0.335 \cdot \text{in}$$

Material Properties: A992

$$F_y := 50 \cdot \text{ksi}$$

$$E := 29000 \cdot \text{ksi}$$

$$G := 11154 \cdot \text{ksi}$$

$$\phi := 0.9$$

$$K := 1.2$$

$$r_y := \sqrt{\frac{I_y}{A}} = 1.954 \text{ in}$$

$$r_z := \sqrt{\frac{I_z}{A}} = 5.154 \text{ in}$$

### Width to Thickness Ratios:

Compression Elements:

$$\frac{b}{t_f} = 7 < 0.56 \cdot \sqrt{\frac{E}{F_y}} = 13.487 \quad \text{Non-Slender Flange (per Table B4.1a, Case 1)}$$

$$\frac{h}{t_w} = 29.6 < 1.49 \cdot \sqrt{\frac{E}{F_y}} = 35.884 \quad \text{Non-Slender Web (per Table B4.1a, Case 5)}$$

Bending Elements:

$$\frac{b}{t_f} = 7 < 0.38 \cdot \sqrt{\frac{E}{F_y}} = 9.152 \quad \text{Compact Flange (per Table B4.1b, Case 10)}$$

$$\frac{h}{t_w} = 29.6 < 3.76 \cdot \sqrt{\frac{E}{F_y}} = 90.553 \quad \text{Compact Web (per Table B4.1b, Case 15)}$$

### Applied Loading per RISA Analysis:

Governing Location: 144 inches

$$P := 3.185 \cdot \text{kip}$$

$$M_z := 88.893 \cdot \text{kip} \cdot \text{ft}$$

$$M_y := 0 \cdot \text{kip} \cdot \text{ft}$$

Axial load at governing location

z-z Moment at governing location

y-y Moment at governing location

### Member M20, Load Combination 13, continued

#### Loading (continued):

$$M_{max} := 88.893 \cdot \text{kip} \cdot \text{ft}$$

Maximum moment for Cb calculation

$$M_A := 22.627 \cdot \text{kip} \cdot \text{ft}$$

Moment at first quarter point for Cb calculation

$$M_B := 10.136 \cdot \text{kip} \cdot \text{ft}$$

Moment at halfway point for Cb calculation

$$M_C := 47.309 \cdot \text{kip} \cdot \text{ft}$$

Moment at third quarter point for Cb calculation

#### Compressive Capacity:

$$L_c := K \cdot L = 14.4 \text{ ft}$$

$$F_{e\_fb} := \frac{(\pi^2 \cdot E)}{\left(\frac{L_c}{r_y}\right)^2} = 36.585 \text{ ksi} \quad (\text{EQN E3-4})$$

$$F_{e\_ftb} := \left( \frac{\pi^2 \cdot E \cdot C_w}{L_c^2} + G \cdot J \right) \cdot \left( \frac{1}{I_z + I_y} \right) = 38.948 \text{ ksi} \quad (\text{EQN E4-2})$$

$$F_e := \min(F_{e\_fb}, F_{e\_ftb}) = 36.585 \text{ ksi}$$

$$\frac{L_c}{r_y} = 88.449 < 4.71 \cdot \sqrt{\frac{E}{F_y}} = 113.432$$

$$F_{cr} := (0.658)^{\left(\frac{F_y}{F_e}\right)} \cdot F_y = 28.219 \text{ ksi} \quad (\text{EQN E3-2})$$

$$P_n := F_{cr} \cdot A = 369.673 \text{ kip} \quad (\text{EQN E3-1})$$

$$P_c := \phi \cdot P_n = 332.706 \text{ kip}$$

#### Flexural Capacity:

##### Yielding-

$$M_{ny\_y} := F_y \cdot Z_y = 79.167 \text{ kip} \cdot \text{ft} \quad (\text{EQN F2-1})$$

$$M_{nz\_y} := F_y \cdot Z_z = 267.5 \text{ kip} \cdot \text{ft} \quad (\text{EQN F6-1})$$

##### Lateral Torsional Buckling-

$$L_p := 1.76 \cdot r_y \cdot \sqrt{\frac{E}{F_y}} = 6.901 \text{ ft} \quad (\text{EQN F2-5})$$

$$L_b := L = 12 \text{ ft}$$

$$L_r := \left( \frac{1.95 \cdot r_{ts} \cdot E}{0.7 \cdot F_y} \right) \cdot \sqrt{\left( \frac{J \cdot c}{S_z \cdot h_o} \right) + \left( \sqrt{\left( \frac{J \cdot c}{S_z \cdot h_o} \right)^2 + 6.76 \cdot \left( \frac{0.7 \cdot F_y}{E} \right)^2} \right)} = 22.402 \text{ ft} \quad (\text{EQN F2-6})$$

$$M_{py} := M_{ny\_y} = 79.167 \text{ kip} \cdot \text{ft}$$

$$M_{pz} := M_{nz\_y} = 267.5 \text{ kip} \cdot \text{ft}$$

**Member M20, Load Combination 13, continued**

---

$$C_b := \frac{12.5 \cdot M_{max}}{2.5 \cdot M_{max} + 3 \cdot M_A + 4 \cdot M_B + 3 \cdot M_C} = 2.351 \quad (\text{EQN F1-1})$$

$$M_{nz\_ltb} := C_b \cdot \left( M_{pz} - (M_{pz} - 0.7 \cdot F_y \cdot S_z) \cdot \left( \frac{L_b - L_p}{L_r - L_p} \right) \right) = 552.224 \text{ kip} \cdot \text{ft} \quad (\text{EQN F2-2})$$

Therefore,  $M_{ny} := M_{ny\_y} = 79.167 \text{ kip} \cdot \text{ft}$   
 $M_{nz} := \min(M_{nz\_y}, M_{nz\_ltb}) = 267.5 \text{ kip} \cdot \text{ft}$   
 $\phi \cdot M_{ny} = 71.25 \text{ kip} \cdot \text{ft}$   
 $\phi \cdot M_{nz} = 240.75 \text{ kip} \cdot \text{ft}$

**Unity Code Check (UC Max):**

$$\frac{P}{P_c} = 0.01 < 0.2$$

Therefore,  $UC_{Max} := \left( \frac{P}{2 \cdot P_c} \right) + \left( \frac{M_z}{\phi \cdot M_{nz}} \right) + \left( \frac{M_y}{\phi \cdot M_{ny}} \right) = 0.374 \quad (\text{EQN H1-1b})$

### Member M16, Load Combination 15:

Shape Properties: WT18x85

$$A := 25 \cdot \text{in}^2$$

$$L := 120 \cdot \text{in}$$

$$I_y := 160 \cdot \text{in}^4$$

$$I_z := 786 \cdot \text{in}^4$$

$$Z_y := 41.8 \cdot \text{in}^3$$

$$Z_z := 105 \cdot \text{in}^3$$

$$S_y := 26.6 \cdot \text{in}^3$$

$$S_z := 58.9 \cdot \text{in}^3$$

$$J := 7.51 \cdot \text{in}^4$$

$$C_w := 63.2 \cdot \text{in}^6$$

$$r_o := 7.437 \cdot \text{in}$$

$$y_{\text{bar}} := 4.73 \cdot \text{in}$$

$$x_o := 0 \cdot \text{in}$$

$$y_o := 4.18 \cdot \text{in}$$

$$d := 18.1 \cdot \text{in}$$

$$b := 6 \cdot \text{in}$$

$$t_f := 1.1 \cdot \text{in}$$

$$t_w := 0.68 \cdot \text{in}$$

Material Properties: A36 Gr.36

$$F_y := 36 \cdot \text{ksi}$$

$$E := 29000 \cdot \text{ksi}$$

$$G := 11154 \cdot \text{ksi}$$

$$\phi := 0.9$$

$$K := 1.2$$

$$r_y := \sqrt{\frac{I_y}{A}} = 2.53 \text{ in}$$

$$r_z := \sqrt{\frac{I_z}{A}} = 5.607 \text{ in}$$

### Width to Thickness Ratios:

Compression Elements:

$$\frac{b}{t_f} = 5.455 < 0.56 \cdot \sqrt{\frac{E}{F_y}} = 15.894 \quad \text{Non-Slender Flange (per Table B4.1a, Case 1)}$$

$$\frac{d}{t_w} = 26.618 > 0.75 \cdot \sqrt{\frac{E}{F_y}} = 21.287 \quad \text{Slender Web (per Table B4.1a, Case 4)}$$

Bending Elements:

$$\frac{b}{t_f} = 5.455 < 0.38 \cdot \sqrt{\frac{E}{F_y}} = 10.785 \quad \text{Compact Flange (per Table B4.1b, Case 10)}$$

$$\frac{d}{t_w} = 26.618 < 1.52 \cdot \sqrt{\frac{E}{F_y}} = 43.141 \quad \text{Non-Compact Web (per Table B4.1b, Case 14)}$$

### Applied Loading (including Torsion) per RISA Analysis:

Governing Location: 0 inches

$$P := 5.7481 \cdot \text{kip}$$

$$\sigma_{bz\_top} := 1.9767 \cdot \text{ksi}$$

$$\sigma_{by\_bot} := 35.9328 \cdot \text{ksi}$$

$$\sigma_{\omega z\_top} := 0 \cdot \text{ksi}$$

Axial load at governing location

Local positive z bending stress at governing location

Local positive y bending stress at governing location

Local top warping bending stress (per Member Torsion spreadsheet) at governing location

## Member M16, Load Combination 15, continued

$$My := (\sigma_{bz\_top} + \sigma_{\omega z\_top}) Sy = 4.382 \text{ kip} \cdot \text{ft}$$

y-y Moment at governing location

$$Mz := \sigma_{by\_bot} \cdot Sz = 176.37 \text{ kip} \cdot \text{ft}$$

z-z Moment at governing location

### Compressive Capacity:

$$\lambda := \frac{d}{tw} = 26.618$$

Slender compression web width to thickness ratio per Table B4.1a (case 4)

$$\lambda_r := 0.75 \cdot \sqrt{\frac{E}{F_y}} = 21.287$$

Limiting width to thickness ratio per Table B4.1a (case 4)

$$Lc := K \cdot L = 12 \text{ ft}$$

$$Fe\_E3 := \frac{\pi^2 \cdot E}{\left(\frac{Lc}{ry}\right)^2} = 88.339 \text{ ksi}$$

(EQN E3-4)

$$Fey := \frac{\pi^2 \cdot E}{\left(\frac{Lc}{ry}\right)^2} = 88.339 \text{ ksi}$$

(EQN E4-6)

$$Fez := \left( \frac{\pi^2 \cdot E \cdot \text{in}^6}{Lc^2} + G \cdot J \right) \cdot \frac{1}{A \cdot ro^2} = 60.591 \text{ ksi}$$

(EQN E4-7) Note Cw is omitted for WT per User Note on page 16.1-37

$$H := 1 - \frac{x_o^2 + y_o^2}{ro^2} = 0.684$$

(EQN E4-8)

$$Fe\_E4 := \left( \frac{Fey + Fez}{2 \cdot H} \right) \cdot \left( 1 - \sqrt{1 - \frac{4 \cdot Fey \cdot Fez \cdot H}{(Fey + Fez)^2}} \right) = 45.413 \text{ ksi}$$

(EQN E4-3)

$$Fe := \min(Fe\_E3, Fe\_E4) = 45.413 \text{ ksi}$$

$$\frac{F_y}{Fe} = 0.793 < 2.25$$

$$Fn := \left( 0.658 \cdot \frac{F_y}{Fe} \right) \cdot F_y = 25.835 \text{ ksi}$$

(EQN E3-2)

$$\lambda = 26.618 > \lambda_r \cdot \sqrt{\frac{F_y}{Fn}} = 25.128$$

$$c1 := 0.22$$

Effective width imperfection adjustment factors per Table E7.1

$$c2 := \frac{1 - \sqrt{1 - 4 \cdot c1}}{2 \cdot c1} = 1.485$$

(EQN E7-4)

$$Fel := \left( c2 \cdot \frac{\lambda_r}{\lambda} \right)^2 \cdot Fey = 50.803 \text{ ksi}$$

(EQN E7-5)

$$de := d \cdot \left( 1 - c1 \cdot \sqrt{\frac{Fel}{Fn}} \right) \cdot \sqrt{\frac{Fel}{Fn}} = 17.551 \text{ in}$$

(EQN E7-3)

### Member M16, Load Combination 15, continued

$$Ae := A - ((d - de) \cdot tw) = 24.627 \text{ in}^2$$

Summation of effective areas based on the reduced effective width, be

$$Pn := Fcr \cdot Ae = 636.231 \text{ kip}$$

(EQN E7-1)

$$Pnc := \phi \cdot Pn = 572.608 \text{ kip}$$

#### Flexural Capacity:

##### Yielding-

$$Mny\_y := \min(Fy \cdot Zy, 1.6 \cdot Fy \cdot Sy) = 125.4 \text{ kip} \cdot \text{ft} \quad (\text{EQN F9-4})$$

$$Mnz\_y := \min(Fy \cdot Zz, 1.6 \cdot Fy \cdot Sz) = 282.72 \text{ kip} \cdot \text{ft} \quad (\text{EQN F9-4})$$

##### Lateral Torsional Buckling-

$$B := -2.3 \cdot \left(\frac{d}{L}\right) \cdot \sqrt{\frac{I_y}{J}} = -1.601 \quad (\text{EQN F9-12})$$

$$Mcr := \frac{1.95 \cdot E}{L} \cdot \sqrt{I_y \cdot J} \cdot (B + \sqrt{1 + B^2}) = 390.149 \text{ kip} \cdot \text{ft} \quad (\text{EQN F9-13})$$

$$Mnz\_ltb := \min(Mcr, Fy \cdot Sz) = 176.7 \text{ kip} \cdot \text{ft} \quad (\text{EQN F9-4})$$

##### Flange Local Buckling-

The flange is compact and in compression, so this check does not apply.

##### Local Buckling of Tee Stems in Flexural Compression-

$$0.84 \cdot \sqrt{\frac{E}{Fy}} = 23.841 < \frac{d}{tw} = 26.618 < 1.52 \cdot \sqrt{\frac{E}{Fy}} = 43.141$$

$$Fcr\_b := \left(1.43 - 0.515 \cdot \left(\frac{d}{tw}\right) \cdot \sqrt{\frac{Fy}{E}}\right) \cdot Fy = 34.093 \text{ ksi} \quad (\text{EQN F9-18})$$

$$Mnz\_lb := Fcr\_b \cdot Sz = 167.338 \text{ kip} \cdot \text{ft} \quad (\text{EQN F9-16})$$

Therefore,  $Mny := Mny\_y = 125.4 \text{ kip} \cdot \text{ft}$

$$Mnz := \min(Mnz\_y, Mnz\_ltb, Mnz\_lb) = 167.338 \text{ kip} \cdot \text{ft}$$

$$\phi \cdot Mny = 112.86 \text{ kip} \cdot \text{ft}$$

$$\phi \cdot Mnz = 150.605 \text{ kip} \cdot \text{ft}$$

#### Unity Code Check (UC Max):

$$\frac{P}{Pnc} = 0.01 < 0.2$$

$$\text{Therefore, } UC\_Max := \left(\frac{P}{2 \cdot Pnc}\right) + \left(\frac{Mz}{\phi \cdot Mnz}\right) + \left(\frac{My}{\phi \cdot Mny}\right) = 1.215 \quad (\text{EQN H1-1b})$$

## LRFD Results Comparison

LRFD Unity Check Comparisons				
Member	Load Combination	RISA-3D	Hand Calculations	% Difference
M10	10	0.058	0.058	0.00
M1	11	0.783	0.783	0.00
M14	12	3.913	3.913	0.00
M25	11	0.174	0.174	0.00
M20	13	0.374	0.374	0.00
M16	15	1.215	1.215	0.00

Table 5.2- LRFD Comparisons

## Conclusion

As can be seen in the chart above, the results match exactly.



# Verification Problem 6

---

## Problem Statement

This problem is a spiral staircase model solved using both RISA-3D and GTStrudl. The structure is a series of short concrete steps, modeled as beams (see Figure 6.1). Uniform loads and self-weight are applied.

The primary use of this problem is to validate RISA-3D against an accepted program other than SAPIV. RISA-3D, SAPIV, and GTStrudl were independently developed and thus can be validated against one another. SAPIV and GTStrudl were both originally developed as mainframe programs using the FORTRAN language, while RISA-3D has been developed as a microcomputer application using the C language.

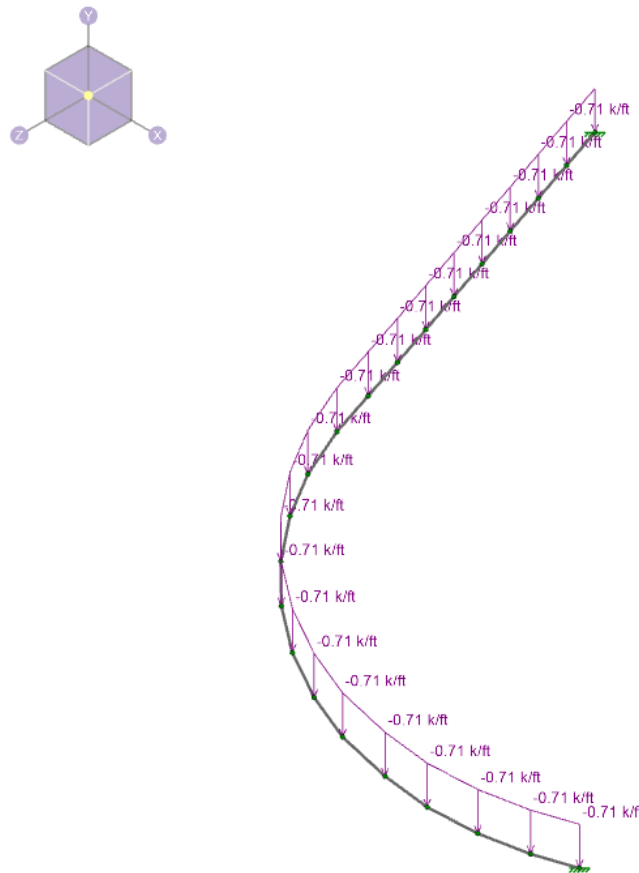


Figure 6.1 – Model Sketch

## Validation Method

The member forces calculated by RISA-3D are compared with the GTStrudl member forces (see Table 6.1). If the member forces match, it is reasonable to assume the joint displacements also match since the member forces are derived from the joint displacements.

## Comparison

Force Comparison: RISA-3D vs. GTStrudl				
Member	Force	RISA-3D Result	GTStrudl Result	% Difference
M1	Axial (k)	20.62	20.62	0.00
M5	Y-Shear (k)	8.94	8.94	0.00
M7	Z-Shear (k)	-14.88	-14.88	0.00
M10	Torque (k-ft)	-0.19	-0.19	0.00
M15	My (k-ft)	-29.73	-29.73	0.00
M18	Mz (k-ft)	2.14	2.14	0.00

Table 6.1 – Force Comparison

## Conclusion

As seen above, the results match exactly.

# Verification Problem 7

---

## Problem Statement

This problem is designed to test the dynamic solution. The first ten frequencies for a simply supported beam, modeled as a series of 50 individual beam elements (see Figure 7.1), are calculated. The beam is also modeled with nearly identical stiffness properties for its y-y and z-z bending axes ( $I_{yy} = 20,000 \text{ in}^4$  &  $I_{zz} = 20,000.1 \text{ in}^4$ ). This means each frequency calculated by the Eigensolver should be duplicated (once for each bending axis). So, to get the first ten separate frequencies, we ask for 19 frequencies to be calculated.

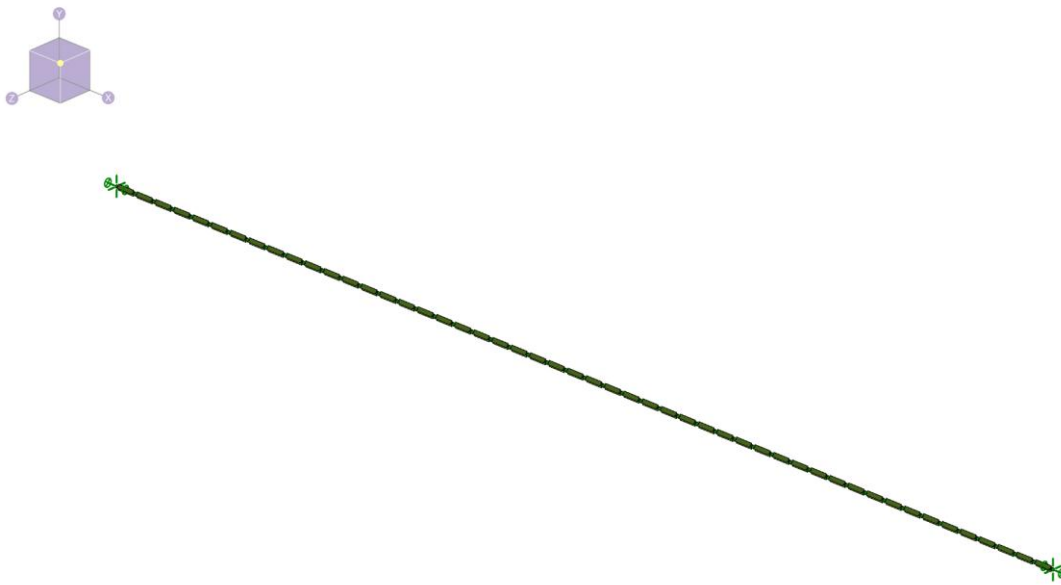


Figure 7.1 – Model Sketch

## Validation Method

The frequencies calculated by RISA-3D will be compared to the “exact” frequencies presented by Formulas for Natural Frequency and Mode Shape by Dr. Robert D. Blevins (see Table 7.1).

The equation presented by Blevins for the transverse frequencies is:

$$F_i = \left( \frac{\Gamma^2}{2 * \pi * L^2} \right) * \sqrt{\frac{E * I}{m}}$$

The equation presented by Blevins for the longitudinal frequencies is:

$$F_i = \left( \frac{\Gamma}{2 * \pi * L} \right) * \sqrt{\frac{E}{\mu}}$$

Where:  $\Gamma = i \cdot \pi$   
 $m$  = mass per unit  
 $\mu$  = mass density  
 $i$  = frequency number ( $i = 1, 2, 3 \dots$ )

For our model:  $E = 30,000$  ksi  
 $I = 20,000$  in<sup>4</sup>  
 $m = 0.10783$  slugs/in  
 $\mu = 0.00074885$  slugs/in<sup>3</sup>

## Comparison

Frequency Comparison: RISA-3D vs. Blevins					
Frequency No.	Blevins Value (Hz)	RISA-3D y-y Axis Values (Hz)	% Difference	RISA-3D z-z Axis Values (Hz)	% Difference
1	0.643	0.643	0.000	0.643	0.000
2	2.573	2.573	0.000	2.573	0.000
3	5.790	5.789	0.000	5.789	0.017
4	10.292	10.292	0.000	10.292	0.000
5	16.085	16.082	0.019	16.082	0.019
6	23.158	23.158	0.000	23.158	0.000
7	31.521	31.520	0.003	31.520	0.003
8	41.170	41.168	0.005	41.168	0.005
9	41.699	41.692	0.017	-	-
10	52.106	52.101	0.010	52.101	0.010

Table 7.1 – Frequency Comparison

\*Note: Frequency No. 9 is the first longitudinal frequency, it appears only once; it is not duplicated.

## Conclusion

As can be seen above, the results match almost exactly.

# Verification Problem 8

---

## Problem Statement

This problem is used to test plate/shell elements for bending, membrane action and “twist.” The problem also gives a verification of a rectangular beam member for torsion. The model is of two cantilever beams, the first modeled using a mesh of finite elements, and the second modeled using a rectangular beam (see Figure 8.1). Three different loadings applied at the free ends of the cantilevers are considered. These are an out-of-plane bending load, an in-plane, vertical membrane load, and a torsional twisting moment.

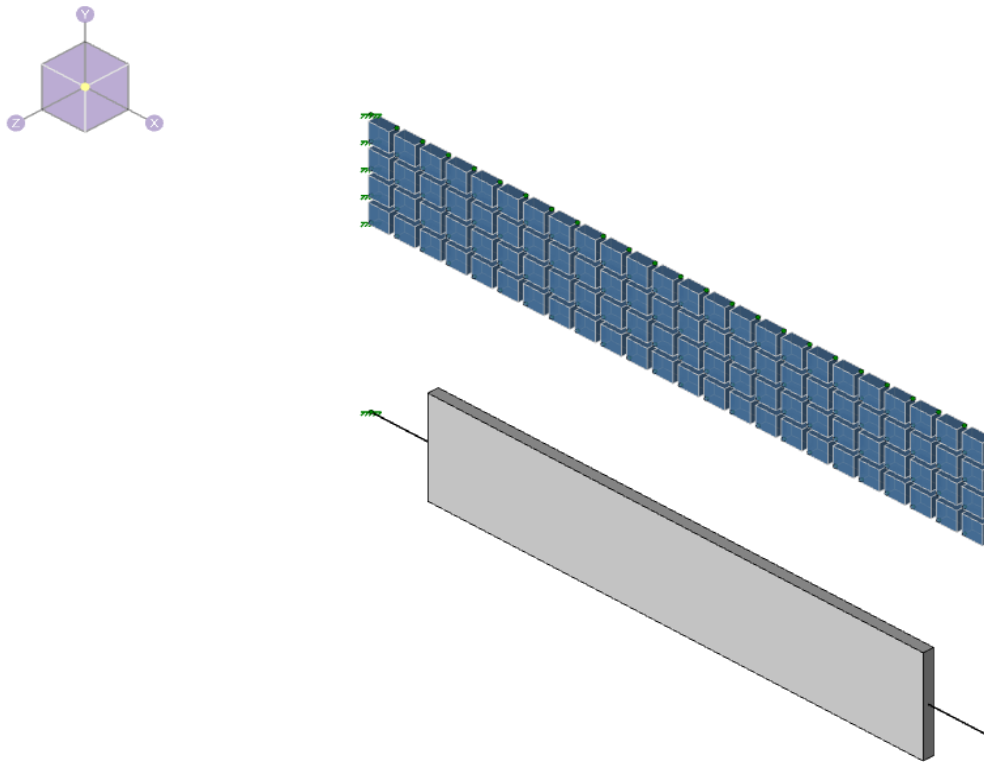


Figure 8.1 – Model Sketch

## Validation Method

This model is validated by comparing the deflections and rotations at the free ends of each cantilever (see Table 8.1). These results will also be checked against theoretical hand calculations. Following are these calculations:

Property Values:

Beam Depth (D)	= 60 in
Beam Width (B)	= 6 in
Area (A)	= 360 in <sup>2</sup>
Length (L)	= 30 ft
Young's Modulus (E)	= 4000 ksi
Shear Modulus (G)	= 1539 ksi
Bending load applied at the free end (P <sub>b</sub> )	= 50 kips
Membrane load applied at the free end (P <sub>m</sub> )	= 5000 kips
Torsional load applied at the free end (T)	= 625 k-ft (7500 k-in)
Moment of Inertia for the Bending Load (I <sub>b</sub> )	= 1080 in <sup>4</sup>
Moment of Inertia for the Membrane Load (I <sub>m</sub> )	= 108,000 in <sup>4</sup>

The torsional stiffness (J) is given by:

For: 2a = D = 60 in    a = 30 in

2b = B = 6 in    b = 3 in

$$J = a * b^3 \left[ \left( \frac{16}{3} \right) - 3.36 * \left( \frac{b}{a} \right) * \left( 1 - \frac{b^4}{12 * a^4} \right) \right] = 4047.8 \text{ in}^4$$

Therefore, for the given property values:

The free end deflection due to the bending load is:

$$\Delta_b = \left[ \left( \frac{P * L^3}{3 * E * I} \right) + \left( \frac{12 * P * L}{A * G} \right) \right] = 180.038 \text{ in}$$

The free end deflection due to the membrane load is:

$$\Delta_m = \left[ \left( \frac{P * L^3}{3 * E * I} \right) + \left( \frac{12 * P * L}{A * G} \right) \right] = 183.899 \text{ in}$$

The free end rotation due to the torsional load is:

$$\Delta = \left( \frac{T * L}{G * J} \right) = 0.43356 \text{ rad}$$

## Comparison

Free End Deflection Comparison: Plates vs. Beams			
Loading	Plates/Shells (Node N8)	Beam (Node N2)	Theory
Bending (Z)	177.042 in	180.038 in	180.038 in
Membrane (Y)	177.470 in	183.825 in	183.899 in
Torsion (X Rot.)	0.402 rad	0.434 rad	0.434 rad

Table 8.1 – Deflection Comparison

## Conclusion

As can be seen above, the results match very closely.

# Verification Problem 9

---

## Problem Statement

This problem is used to test the Dynamic Analysis and the Response Spectrum Analysis (RSA) features in RISA-3D. The model for this problem is essentially a flagpole with asymmetric triangular projections at five elevations (see Fig. 9.1). The asymmetric projections of the “flagpole” will ensure that there is a large amount of modal coupling between the lateral modes. This is desirable because it will highlight any errors in the SRSS spatial combination. A model with no modal coupling will give the same spatially combined spectral results using the SRSS rule or an absolute sum.

The model will be analyzed in all three global directions using the CQC modal combination method with 5% damping. These spectral results will be added using the SRSS spatial combination option and then compared to the results of the same model in SAP2000. The three separate results will also be combined as an absolute sum and compared to the results of the SRSS reactions.

The 1991/94 UBC design spectra for soil type S1 will be the response spectra used to obtain the spectral results. Multipliers were applied to the S1 spectra as follows: 1.0 for the SX, 0.5 for the SY, and 0.3 for the SZ. The mass used for the dynamic solution consists of concentrated loads to all the free joints. Self-weight was not included in the model solution.

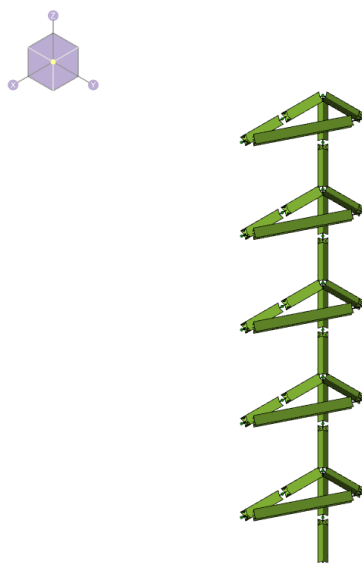


Figure 9.1 – Model Sketch

## Validation Method

The model was built as shown above made up of rectangular steel sections with the J value assumed to equal 182.52 in<sup>4</sup>. The frequencies, mass participation factors, the reaction at the free end, and the spectral displacements at the tip of the upper triangle will be calculated by RISA-3D and then compared against the same model run in SAP2000 (see Tables 9.1-9.4).

The comparison of the frequencies and the mass participation will be to check the dynamic solution and RSA. The reactions at the fixed end and the displacements at the top triangle tip will check the RSA and the SRSS combination feature.



## Comparison

Frequencies and Mass Participation Factors by Mode								
Mode	RISA-3D Results				SAP2000 Results			
	Freq. (Hz)	Mass Participation (%)			Freq. (Hz)	Mass Participation (%)		
		SX	SY	SZ		SX	SY	SZ
1	0.44	47.60	16.93	0.64	0.44	47.59	16.94	0.64
2	0.444	16.15	49.37	0.85	0.44	16.16	49.37	0.85
3	1.891	0.41	1.73		1.89	0.41	1.73	
4	2.488	18.47	0.04	1.36	2.49	18.48	0.04	1.36
5	2.673	0.14	18.14	0.27	2.67	0.14	18.14	0.27
6	5.117	0.94	1.29		5.12	0.94	1.29	
7	5.947	4.11	0.35	0.91	5.94	4.11	0.35	0.91
8	6.555	0.02	3.83	0.03	6.55	0.02	3.82	0.03
9	7.757	0.48	0.39		7.75	0.46	0.39	
10	8.775	1.05	0.31	1.03	8.77	1.05	0.31	1.03
11	9.188	0.22	0.08	0.12	9.18	0.22	0.07	0.12
12	10.306	0.25	0.08		10.30	0.25	0.08	
13	10.548	0.03	1.93	0.12	10.54	0.03	1.93	0.12
14	12.893	3.61		26.53	12.87	3.61		26.46
15	14.046	1.96		9.94	14.02	1.95		9.99
16	16.083	0.49	1.14	0.51	16.06	0.50	1.12	0.51
17	16.918	1.03	0.30	0.06	16.88	1.01	0.31	0.05
18	20.895	1.18	0.10	1.78	20.84	1.18	0.10	1.78
19	22.374	0.13	0.47		22.34	0.12	0.48	
20	25.696	0.46	0.18	0.99	25.61	0.45	0.18	0.98
21	28.873	0.06	1.53	15.94	28.78	0.06	1.56	15.44
22	29.56	0.02	0.73	15.41	29.48	0.01	0.69	15.81
23	33.963		0.01	1.00	33.83		0.01	0.99
24	34.94		0.01	0.32	34.80		0.01	0.33
25	36.202	0.02	0.02	0.04	36.06	0.02	0.01	0.04
26	52.375			14.81	52.26			14.92
27	66.964	0.07		0.01	66.63	0.07		0.01
28	73.013	0.17		0.11	72.59	0.17		0.11
29	79.308	0.10			75.76	0.10		0.01
30	81.552	0.06		1.11	80.96	0.05		1.10
Total	--	99.17	98.94	93.92	--	99.16	98.93	93.86

Table 9.1 – Frequencies and Mass Participation Factors

As can be seen in the chart above, the frequencies and mass participation factors match almost exactly for all modes.

Comparison of the Fixed End Spectral Reactions							
Program	Node	RX (k)	RY (k)	RZ (k)	MX (k-ft)	MY (k-ft)	MZ (k-ft)
RISA-3D	N1	55.75	28.42	30.82	251.62	497.88	41.14
SAP2000	N3	55.94	28.52	30.82	254.30	502.90	41.50
% Difference	--	0.34	0.34	0.00	1.06	1.00	0.86

Table 9.2 – Spectral Reactions

Note: The signs of the RISA results have been adjusted to match SAP2000 sign convention

These reactions were obtained from the SRSS combination of all three spectral results (SX,SY,and SZ). As shown above, the reactions at the fixed end are also almost identical.

Comparison of the Top Level Deflections (at the Tip of the Flagpole Projection)							
Program	Node	X (in)	Y (in)	Z (in)	θX (rad)	θY (rad)	θZ (rad)
RISA-3D	N21	29.36	15.97	8.75	0.09	0.18	0.05
SAP2000	N78	29.79	16.17	8.85	0.09	0.18	0.05
% Difference	--	1.44	1.24	1.13	0.00	0.00	0.00

Table 9.3 – Tip Deflections

These reactions were obtained from the SRSS combination of all three spectral results (SX, SY, and SZ). As shown above, the deflections at the tip of the top level are almost exactly the same.

Absolute Sum Spatial Combination of the SX, SY, and SZ RSA's							
Program	Node	RX (k)	RY (k)	RZ (k)	MX (k-ft)	MY (k-ft)	MZ (k-ft)
RISA-3D	N1	64.05	35.08	46.60	289.98	540.80	59.42

Table 9.4 – Spatial Combination

Note: The signs of the RISA results have been adjusted to match SAP2000 sign convention

The chart above shows all three spectral reactions (in absolute terms) from RISA-3D combined together as an absolute sum. This is included in order to compare the results to those of the SRSS spatial combination. As can be seen, the reactions are quite a bit larger than those from the SRSS combination calculation.

# Verification Problem 10

---

## Problem Statement

This problem tests the *ANSI/AWC NDS-2015* ASD code check. The two bay portal frame model (see Fig. 10.1) is made up of several different shapes, species, and grades of lumber, with one bay braced in the X-direction. The model is loaded with combinations of Dead Load, Live Load, and Lateral (Wind) Load. A different CD (Load Duration) factor is used for each load combination.

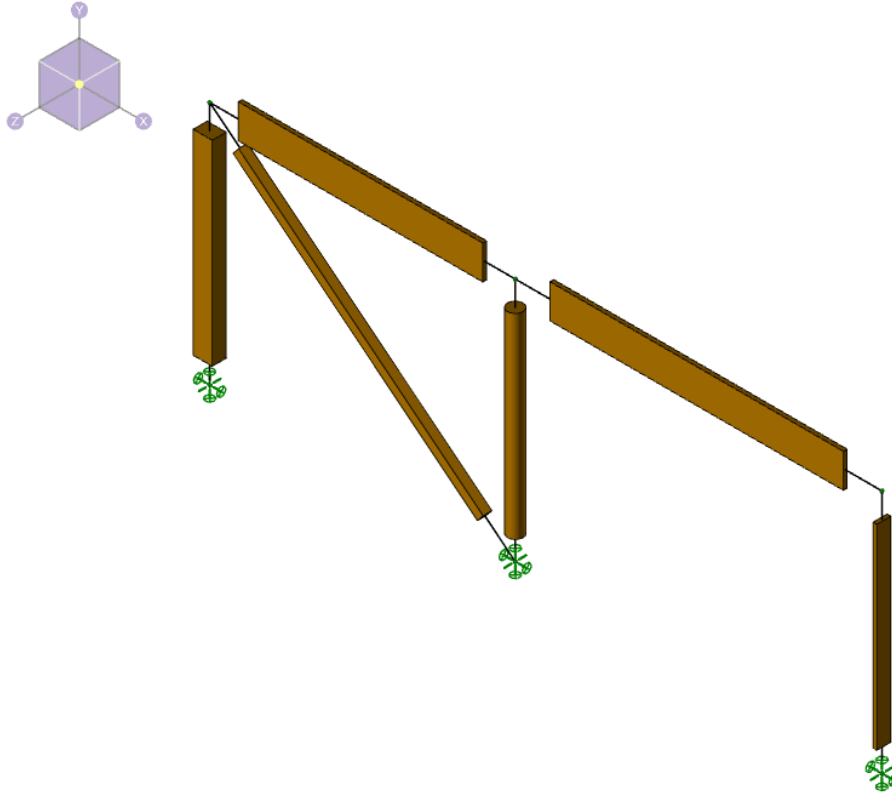


Figure 10.1- Model Sketch

## Validation Method

Following are the hand calculations for various members for various load combinations. All code check calculations and wood properties are from the *ANSI/AWC NDS-2015* including the Supplement (see Table 10.1). Several different situations commonly encountered in wood design are shown here, such as columns, beams, and combined beam/column members. The member stresses (axial, bending, and shear) will also be calculated as part of the verification.

### Member M1, Load Combo 3: (DL +LL+Wind)

#### Input & Analysis Values:

Shape Properties (6x8):

$$b := 5.5 \cdot \text{in}$$

$$d := 7.5 \cdot \text{in}$$

$$A := b \cdot d = 41.25 \text{ in}^2$$

$$L_z := 96 \cdot \text{in}$$

$$L_y := 96 \cdot \text{in}$$

$$S_z := \frac{b \cdot d^2}{6} = 51.563 \text{ in}^3$$

$$S_y := \frac{d \cdot b^2}{6} = 37.813 \text{ in}^3$$

Material Properties (C1: DF-Larch, No. 1 Dense):

$$E := 1700000 \cdot \text{psi}$$

$$F_b := 1400 \cdot \text{psi}$$

$$F_t := 950 \cdot \text{psi}$$

$$F_v := 170 \cdot \text{psi}$$

$$F_c := 1200 \cdot \text{psi}$$

Design Forces (from the RISA analysis):

$$P := 3965.447 \cdot \text{lbf}$$

Axial force (Tension) at governing location (48 in)

$$M_z := 2400 \cdot \text{ft} \cdot \text{lbf}$$

Strong axis bending moment at governing location (48 in)

$$M_y := 0 \cdot \text{lbf} \cdot \text{ft}$$

Weak axis bending moment at governing location (48 in)

$$V := 1200 \cdot \text{lbf}$$

Shear force at governing location (0 in)

Design Stresses (from the RISA analysis):

$$f_a := \frac{P}{A} = 96.132 \text{ psi}$$

Axial stress per LC3 (Tension)

$$f_{bz} := \frac{M_z}{S_z} = 558.5455 \text{ psi}$$

Strong axis bending stress per LC3

$$f_{by} := \frac{M_y}{S_y} = 0 \text{ psi}$$

Weak axis bending stress per LC3

$$f_v := \frac{3 \cdot V}{2 \cdot A} = 43.636 \text{ psi}$$

Shear Stress per LC3

#### Design Calculations:

Load Factors (per input variables):

$$C_D := 1.6$$

Load Duration Factor per Design tab of Load Combinations spreadsheet

$$C_r := 1.0$$

Repetitive Member Factor per Wood tab of Members spreadsheet

$$C_{fu} := 1.0$$

Flat Use Factor per member orientation relative to loading

$$C_t := 1.0$$

Temperature Factor per Wood code selection in (Global) Model Settings

$$C_i := 1.0$$

Incising Factor (always assumed as 1.0 by RISA-3D)

$$C_T := 1.0$$

Buckling Stiffness Factor (always assumed as 1.0 by RISA-3D)

$$C_F := 1.0$$

Size Factor per selected member shape and material  
(per NDS Supplement Table 4D)

$$C_{m,c} := 0.91$$

Wet Service Factor per Wood tab of Materials spreadsheet  
(for  $F_c$  calculation per NDS Supplement Table 4D)

$$C_m := 1.0$$

Wet Service Factor per Wood tab of Materials spreadsheet  
(for  $F_b$ ,  $F_t$ ,  $F_v$ , and  $E$  calculations per NDS Supplement Table 4D)

Emin Calculation:

$$COV_E := 0.25$$

Per Table F1 of NDS Appendix F

$$E_{min} := E \cdot \left(1 - 1.645 \cdot COV_E\right) \cdot \left(\frac{1.03}{1.66}\right) = 621024.849 \text{ psi}$$

Per NDS Appendix D Eqn. (D-4)

$$E_{min'} := E_{min} \cdot C_m \cdot C_m \cdot C_m \cdot C_T = 621024.849 \text{ psi}$$

Per NDS Table 4.3.1

Compressive Capacity:

$$\frac{L_z}{d} = 12.8$$

Strong Axis (z-z) slenderness Ratio ( $l_{e1}/d1$ )

$$\frac{L_y}{b} = 17.4545$$

Weak Axis (y-y) slenderness Ratio ( $l_{e2}/d2$ )

$$S := \max\left(\frac{L_z}{d}, \frac{L_y}{b}\right) = 17.455$$

Maximum Slenderness Ratio

$$F_{cE} := \left(\frac{0.822 \cdot E_{min'}}{S^2}\right) = 1675.574 \text{ psi}$$

Per NDS Section 3.7.1

$$F_{c_{star}} := F_c \cdot C_D \cdot C_{m_c} \cdot C_t \cdot C_F \cdot C_i = 1747.2 \text{ psi}$$

Per NDS Section 3.7.1

$$c := 0.8$$

Per NDS Section 3.7.1

$$C_P := \left(\frac{1 + \left(\frac{F_{cE}}{F_{c_{star}}}\right)}{2 \cdot c}\right) - \sqrt{\left(\left(\frac{1 + \left(\frac{F_{cE}}{F_{c_{star}}}\right)}{2 \cdot c}\right)^2 - \left(\frac{\left(\frac{F_{cE}}{F_{c_{star}}}\right)}{c}\right)\right)} = 0.676$$

Per NDS Eqn (3.7-1)

$$E' := E \cdot C_m \cdot C_t \cdot C_i = 1700000 \text{ psi}$$

Per NDS Table 4.3.1

$$F_{c'} := F_c \cdot C_D \cdot C_{m_c} \cdot C_t \cdot C_F \cdot C_i \cdot C_P = 1181.7019 \text{ psi}$$

Per NDS Table 4.3.1

Tensile Capacity:

$$F_t' := F_t \cdot C_D \cdot C_m \cdot C_F \cdot C_i = 1520 \text{ psi}$$

Per NDS Table 4.3.1

Flexural Capacities:

$$R_B := \sqrt{\frac{L_y \cdot d}{b^2}} = 4.879$$

Per NDS Eqn (3.3-5)

$$F_{bE} := \frac{1.2 \cdot E_{min'}}{R_B^2} = 31310.003 \text{ psi}$$

Per NDS Section 3.3.3

$$F_{b_{star}} := F_b \cdot C_D \cdot C_m \cdot C_F \cdot C_i \cdot C_r = 2240 \text{ psi}$$

Per NDS Section 3.3.3

$$C_{Lz} := \left( \frac{1 + \left( \frac{F_{bE}}{F_{b\_star}} \right)}{1.9} \right) - \sqrt{\left( \left( \frac{1 + \left( \frac{F_{bE}}{F_{b\_star}} \right)}{1.9} \right)^2 - \left( \frac{\left( \frac{F_{bE}}{F_{b\_star}} \right)}{0.95} \right) \right)} = 0.9962 \quad \text{Per NDS Eqn (3.3-6)}$$

Per NDS Section 3.3.3.1

$$C_{Ly} := 1.0$$

$$F_{b1'} := F_b \cdot C_D \cdot C_m \cdot C_{Lz} \cdot C_F \cdot C_{fu} \cdot C_i \cdot C_r = 2231.4382 \text{ psi} \quad \text{Per NDS Table 4.3.1}$$

$$F_{b2'} := F_b \cdot C_D \cdot C_m \cdot C_{Ly} \cdot C_F \cdot C_{fu} \cdot C_i \cdot C_r = 2240 \text{ psi} \quad \text{Per NDS Table 4.3.1}$$

#### Shear Capacity:

$$F_v := F_v \cdot C_D \cdot C_m \cdot C_t \cdot C_i = 272 \text{ psi} \quad \text{Per NDS Table 4.3.1}$$

#### Code Check Calculations:

##### Max Bending Check:

$$UC_{Max} := \left( \frac{f_a}{F_{t'}} \right) + \left( \frac{f_{bz}}{F_{b1'}} \right) + \left( \frac{f_{by}}{F_{b2'}} \right) = 0.314 \quad \text{Per NDS Eqn (3.9-1)}$$

##### Max Shear Check:

$$UC_{Shear} := \frac{f_v}{F_v} = 0.16 \quad \text{Actual over allowable}$$

## Member M2, Load Combo 2: (DL +LL)

### Input & Analysis Values:

Shape Properties (6" Round Pole):

$$D := 6 \cdot \text{in}$$

$$A := \pi \cdot \left(\frac{D}{2}\right)^2 = 28.274 \text{ in}^2$$

Per NDS section 3.7.3, the design of a round section will use:

$$b := \sqrt{A} = 5.317 \text{ in}$$

$$d := \sqrt{A} = 5.317 \text{ in}$$

$$L_z := 96 \cdot \text{in}$$

$$L_y := 96 \cdot \text{in}$$

$$L_{\text{bend}} := 48 \cdot \text{in}$$

$$S_z := \frac{b \cdot d^2}{6} = 25.057 \text{ in}^3$$

$$S_y := \frac{d \cdot b^2}{6} = 25.057 \text{ in}^3$$

Material Properties (C2: Hem-Fir, Select Structural):

$$E := 1300000 \cdot \text{psi}$$

$$F_b := 1200 \cdot \text{psi}$$

$$F_t := 800 \cdot \text{psi}$$

$$F_v := 140 \cdot \text{psi}$$

$$F_c := 975 \cdot \text{psi}$$

Design Forces (from the RISA analysis):

$$P := 5515.28 \cdot \text{lbf}$$

$$M_z := 0 \cdot \text{ft} \cdot \text{lbf}$$

$$M_y := 0 \cdot \text{lbf} \cdot \text{ft}$$

$$V := 0 \cdot \text{lbf}$$

Axial force (Compression) at governing location (0 in)

Strong axis bending moment at governing location (0 in)

Weak axis bending moment at governing location (0 in)

Shear force at governing location (0 in)

Design Stresses (from the RISA analysis):

$$f_a := \frac{P}{A} = 195.063 \text{ psi}$$

Axial stress per LC2 (Compression)

$$f_{bz} := \frac{M_z}{S_z} = 0 \text{ psi}$$

Strong axis bending stress per LC2

$$f_{by} := \frac{M_y}{S_y} = 0 \text{ psi}$$

Weak axis bending stress per LC2

$$f_v := \frac{3 \cdot V}{2 \cdot A} = 0 \text{ psi}$$

Shear Stress per LC2

### Design Calculations:

Load Factors (per input variables):

$$C_D := 1.0$$

Load Duration Factor per Design tab of Load Combinations spreadsheet

$$C_r := 1.0$$

Repetitive Member Factor per Wood tab of Members spreadsheet

$$C_{fu} := 1.0$$

Flat Use Factor per member orientation relative to loading

$$C_t := 1.0$$

Temperature Factor per Wood code selection in (Global) Model Settings

$$C_i := 1.0$$

Incising Factor (always assumed as 1.0 by RISA-3D)

$$C_T := 1.0$$

Buckling Stiffness Factor (always assumed as 1.0 by RISA-3D)

$C_F := 1.0$	Size Factor per selected member shape and material (per NDS Supplement Table 4D)
$C_m := 1.0$	Wet Service Factor per Wood tab of Materials spreadsheet (per NDS Supplement Table 4D)

Emin Calculation:

$COV_E := 0.25$	Per Table F1 of NDS Appendix F
$E_{min} := E \cdot \left(1 - 1.645 \cdot COV_E\right) \cdot \left(\frac{1.03}{1.66}\right) = 474901.355 \text{ psi}$	Per NDS Appendix D Eqn. (D-4)
$E_{min}' := E_{min} \cdot C_m \cdot C_m \cdot C_m \cdot C_T = 474901.355 \text{ psi}$	Per NDS Table 4.3.1

Compressive Capacity:

$\frac{L_z}{d} = 18.0541$	Strong Axis (z-z) slenderness Ratio ( $l_e1/d1$ )
$\frac{L_y}{b} = 18.0541$	Weak Axis (y-y) slenderness Ratio ( $l_e2/d2$ )
$S := \max\left(\frac{L_z}{d}, \frac{L_y}{b}\right) = 18.0541$	Maximum Slenderness Ratio
$F_{cE} := \left(\frac{0.822 \cdot E_{min}'}{S^2}\right) = 1197.637 \text{ psi}$	Per NDS Section 3.7.1
$F_{c,star} := F_c \cdot C_D \cdot C_m \cdot C_t \cdot C_F \cdot C_i = 975 \text{ psi}$	Per NDS Section 3.7.1
$c := 0.85$	Per NDS Section 3.7.1
$C_P := \left(\frac{1 + \left(\frac{F_{cE}}{F_{c,star}}\right)}{2 \cdot c}\right) - \sqrt{\left(\left(\frac{1 + \left(\frac{F_{cE}}{F_{c,star}}\right)}{2 \cdot c}\right)^2 - \left(\frac{F_{cE}}{F_{c,star}}\right)\right)} = 0.7882$	Per NDS Eqn (3.7-1)
$E' := E \cdot C_m \cdot C_t \cdot C_i = 1300000 \text{ psi}$	Per NDS Table 4.3.1
$F_c' := F_c \cdot C_D \cdot C_m \cdot C_t \cdot C_F \cdot C_i \cdot C_P = 768.5322 \text{ psi}$	Per NDS Table 4.3.1

Tensile Capacity:

$F_t' := F_t \cdot C_D \cdot C_m \cdot C_F \cdot C_i = 800 \text{ psi}$	Per NDS Table 4.3.1
---	---------------------

Flexural Capacities:

$R_B := \sqrt{\frac{L_{bend} \cdot d}{b^2}} = 3.0045$	Per NDS Eqn (3.3-5)
---	---------------------



$$F_{bE} := \frac{1.2 \cdot E_{min'}}{R_g^2} = 63130.555 \text{ psi} \quad \text{Per NDS Section 3.3.3}$$

$$F_{b_{star}} := F_b \cdot C_D \cdot C_m \cdot C_F \cdot C_i \cdot C_r = 1200 \text{ psi} \quad \text{Per NDS Section 3.3.3}$$

$$C_{Lz} := 1.0 \quad \text{Per NDS Section 3.3.3.1}$$

$$C_{Ly} := 1.0 \quad \text{Per NDS Section 3.3.3.1}$$

$$F_{b1'} := F_b \cdot C_D \cdot C_m \cdot C_{Lz} \cdot C_F \cdot C_{fu} \cdot C_i \cdot C_r = 1200 \text{ psi} \quad \text{Per NDS Table 4.3.1}$$

$$F_{b2'} := F_b \cdot C_D \cdot C_m \cdot C_{Ly} \cdot C_F \cdot C_{fu} \cdot C_i \cdot C_r = 1200 \text{ psi} \quad \text{Per NDS Table 4.3.1}$$

#### Shear Capacity:

$$F_v := F_v \cdot C_D \cdot C_m \cdot C_t \cdot C_i = 140 \text{ psi} \quad \text{Per NDS Table 4.3.1}$$

#### Code Check Calculations:

##### Max Bending Check:

$$UC_{Max} := \left( \frac{f_a}{F_c'} \right) = 0.254 \quad \text{Per NDS Eqn (3.6.3)}$$

##### Max Shear Check:

$$UC_{Shear} := \frac{f_v}{F_v'} = 0 \quad \text{Actual over allowable}$$

\*Note: For some members the limitations in section 3.6.3 control over any of the equations. This is because in the Compression-Bending Interaction equation (Eqn. 3.9-3), if the bending goes to zero, the equation will automatically square the compression portion, lowering it from what we know to be the actual capacity ( $f_c/F_c'$  vs.  $(f_c/F_c')^2$ ). This section allows us to use the compression portion without squaring it to know the true capacity of the compression-only member.

### Member M3, Load Combo 3: (DL +LL+Wind)

#### Input & Analysis Values:

Shape Properties

(2x6, rotated 90 degrees):

$$b := 1.5 \cdot \text{in}$$

$$d := 5.5 \cdot \text{in}$$

$$A := b \cdot d = 8.25 \text{ in}^2$$

$$L_z := 24 \cdot \text{in}$$

$$L_y := 24 \cdot \text{in}$$

$$S_z := \frac{b \cdot d^2}{6} = 7.563 \text{ in}^3$$

$$S_y := \frac{d \cdot b^2}{6} = 2.063 \text{ in}^3$$

Material Properties (C3: Yellow Poplar, No. 1):

$$E := 1400000 \cdot \text{psi}$$

$$F_b := 725 \cdot \text{psi}$$

$$F_t := 425 \cdot \text{psi}$$

$$F_y := 145 \cdot \text{psi}$$

$$F_c := 725 \cdot \text{psi}$$

Design Forces (from the RISA analysis):

$$P := 2107.04 \cdot \text{lbf}$$

Axial force (Compression) at governing location (24 in)

$$M_z := 0 \cdot \text{ft} \cdot \text{lbf}$$

Strong axis bending moment at governing location (24 in)

$$M_y := 750 \cdot \text{lbf} \cdot \text{ft}$$

Weak axis bending moment at governing location (24 in)

$$V := 375 \cdot \text{lbf}$$

Shear force at governing location (0 in)

Design Stresses (from the RISA analysis):

$$f_a := \frac{P}{A} = 255.399 \text{ psi}$$

Axial stress per LC3 (Compression)

$$f_{bz} := \frac{M_z}{S_z} = 0 \text{ psi}$$

Strong axis bending stress per LC3

$$f_{by} := \frac{M_y}{S_y} = 4363.636 \text{ psi}$$

Weak axis bending stress per LC3

$$f_v := \frac{3 \cdot V}{2 \cdot A} = 68.182 \text{ psi}$$

Shear Stress per LC3

#### Design Calculations:

Load Factors (per input variables):

$$C_D := 1.6$$

Load Duration Factor per Design tab of Load Combinations spreadsheet

$$C_r := 1.0$$

Repetitive Member Factor per Wood tab of Members spreadsheet

$$C_{fu} := 1.15$$

Flat Use Factor per member orientation relative to loading

$$C_t := 1.0$$

Temperature Factor per Wood code selection in (Global) Model Settings

$$C_i := 1.0$$

Incising Factor (always assumed as 1.0 by RISA-3D)

$$C_T := 1.0$$

Buckling Stiffness Factor (always assumed as 1.0 by RISA-3D)

$$C_{F_c} := 1.1$$

Size Factor per selected member shape and material  
(for the  $F_c$  calculation per NDS Supplement Table 4A)

$$C_F := 1.3$$

Size Factor per selected member shape and material  
(for the  $F_b$  &  $F_t$  calculations per NDS Supplement Table 4A)

$$C_m := 1.0$$

Wet Service Factor per Wood tab of Materials spreadsheet  
(per NDS Supplement Table 4A)

Emin Calculation:

$$COV_E := 0.25$$

Per Table F1 of NDS Appendix F

$$E_{min} := E \cdot \left(1 - 1.645 \cdot COV_E\right) \cdot \left(\frac{1.03}{1.66}\right) = 511432.229 \text{ psi}$$

Per NDS Appendix D Eqn. (D-4)

$$E_{min}' := E_{min} \cdot C_m \cdot C_m \cdot C_m \cdot C_T = 511432.229 \text{ psi}$$

Per NDS Table 4.3.1

Compressive Capacity:

$$\frac{L_z}{d} = 4.364$$

Strong Axis (z-z) slenderness Ratio ( $l_e1/d1$ )

$$\frac{L_y}{b} = 16$$

Weak Axis (y-y) slenderness Ratio ( $l_e2/d2$ )

$$S := \max\left(\frac{L_z}{d}, \frac{L_y}{b}\right) = 16$$

Maximum Slenderness Ratio

$$F_{cE} := \left(\frac{0.822 \cdot E_{min}'}{S^2}\right) = 1642.177 \text{ psi}$$

Per NDS Section 3.7.1

$$F_{c_{star}} := F_c \cdot C_D \cdot C_m \cdot C_t \cdot C_{F_c} \cdot C_i = 1276 \text{ psi}$$

Per NDS Section 3.7.1

$$c := 0.8$$

Per NDS Section 3.7.1

$$C_P := \left(\frac{1 + \left(\frac{F_{cE}}{F_{c_{star}}}\right)}{2 \cdot c}\right) - \sqrt{\left(\left(\frac{1 + \left(\frac{F_{cE}}{F_{c_{star}}}\right)}{2 \cdot c}\right)^2 - \left(\frac{\left(\frac{F_{cE}}{F_{c_{star}}}\right)}{c}\right)\right)} = 0.7703$$

Per NDS Eqn (3.7-1)

$$E' := E \cdot C_m \cdot C_t \cdot C_i = 1400000 \text{ psi}$$

Per NDS Table 4.3.1

$$F_c' := F_c \cdot C_D \cdot C_m \cdot C_t \cdot C_{F_c} \cdot C_i \cdot C_P = 982.9116 \text{ psi}$$

Per NDS Table 4.3.1

Tensile Capacity:

$$F_t' := F_t \cdot C_D \cdot C_m \cdot C_F \cdot C_i = 884 \text{ psi}$$

Per NDS Table 4.3.1

Flexural Capacities:

$$R_B := \sqrt{\frac{L_y \cdot d}{b^2}} = 7.659$$

Per NDS Eqn (3.3-5)

$$F_{bE} := \frac{1.2 \cdot E_{min}'}{R_B^2} = 10461.114 \text{ psi}$$

Per NDS Section 3.3.3

$$F_{b_{star}} := F_b \cdot C_D \cdot C_m \cdot C_F \cdot C_i \cdot C_r = 1508 \text{ psi}$$

Per NDS Section 3.3.3

$$C_{Lz} := \left( \frac{1 + \left( \frac{F_{bE}}{F_{b\_star}} \right)}{1.9} \right) - \sqrt{\left( \left( \frac{1 + \left( \frac{F_{bE}}{F_{b\_star}} \right)}{1.9} \right)^2 - \left( \frac{F_{bE}}{F_{b\_star}} \right) \right)} = 0.9917$$

Per NDS Eqn (3.3-6)

Per NDS Section 3.3.3.1

$$C_{Ly} := 1.0$$

$$F_{b1'} := F_b \cdot C_D \cdot C_m \cdot C_{Lz} \cdot C_F \cdot C_i \cdot C_r = 1495.5267 \text{ psi}$$

Per NDS Table 4.3.1

$$F_{b2'} := F_b \cdot C_D \cdot C_m \cdot C_{Ly} \cdot C_F \cdot C_{fu} \cdot C_i \cdot C_r = 1734.2 \text{ psi}$$

Per NDS Table 4.3.1

#### Shear Capacity:

$$F_{v'} := F_v \cdot C_D \cdot C_m \cdot C_t \cdot C_i = 232 \text{ psi}$$

Per NDS Table 4.3.1

#### Code Check Calculations:

$$F_{cE1} := \frac{0.822 \cdot E_{min'}}{\left( \frac{L_z}{d} \right)^2} = 22078.156 \text{ psi}$$

Per NDS Section 3.9.2

$$F_{cE2} := \frac{0.822 \cdot E_{min'}}{\left( \frac{L_y}{b} \right)^2} = 1642.177 \text{ psi}$$

Per NDS Section 3.9.2

#### Max Bending Check:

$$UC_{Max} := \left( \frac{f_a}{F_{c'}} \right)^2 + \left( \frac{f_{bz}}{F_{b1'} \cdot \left( 1 - \left( \frac{f_a}{F_{cE1}} \right) \right)} \right) + \left( \frac{f_{by}}{F_{b2'} \cdot \left( 1 - \left( \frac{f_a}{F_{cE2}} \right) - \left( \frac{f_{bz}}{F_{bE}} \right)^2 \right)} \right) = 3.047$$

Per NDS Eqn (3.9-3)

#### Max Shear Check:

$$UC_{Shear} := \frac{f_v}{F_{v'}} = 0.294$$

Actual over allowable

### Member M5, Load Combo 1: (DL Only)

#### Input & Analysis Values:

Shape Properties(2x14):

$$b := 1.5 \cdot \text{in}$$

$$d := 13.25 \cdot \text{in}$$

$$A := b \cdot d = 19.875 \text{ in}^2$$

$$L_z := 144 \cdot \text{in}$$

$$L_y := 60 \cdot \text{in}$$

$$L_{\text{bend}} := 60 \cdot \text{in}$$

$$S_z := \frac{b \cdot d^2}{6} = 43.891 \text{ in}^3$$

$$S_y := \frac{d \cdot b^2}{6} = 4.969 \text{ in}^3$$

Material Properties (BM: Doug Fir-Larch, No. 2):

$$E := 1600000 \cdot \text{psi}$$

$$F_b := 850 \cdot \text{psi}$$

$$F_t := 500 \cdot \text{psi}$$

$$F_v := 180 \cdot \text{psi}$$

$$F_c := 1400 \cdot \text{psi}$$

Design Forces (from the RISA analysis):

$$P := 0 \cdot \text{lbf}$$

Axial force at governing location (82.5 in)

$$M_z := 5964.169 \cdot \text{ft} \cdot \text{lbf}$$

Strong axis bending moment at governing location (82.5 in)

$$M_y := 0 \cdot \text{lbf} \cdot \text{ft}$$

Weak axis bending moment at governing location (82.5 in)

$$V := 3034.35 \cdot \text{lbf}$$

Shear force at governing location (0 in)

Design Stresses (from the RISA analysis):

$$f_a := \frac{P}{A} = 0 \text{ psi}$$

Axial stress per LC1

$$f_{bz} := \frac{M_z}{S_z} = 1630.645 \text{ psi}$$

Strong axis bending stress per LC1

$$f_{by} := \frac{M_y}{S_y} = 0 \text{ psi}$$

Weak axis bending stress per LC1

$$f_v := \frac{3 \cdot V}{2 \cdot A} = 229.008 \text{ psi}$$

Shear Stress per LC1

#### Design Calculations:

Load Factors (per input variables):

$$C_D := 0.9$$

Load Duration Factor per Design tab of Load Combinations spreadsheet

$$C_r := 1.0$$

Repetitive Member Factor per Wood tab of Members spreadsheet

$$C_{fu} := 1.2$$

Flat Use Factor per member orientation relative to loading

$$C_t := 1.0$$

Temperature Factor per Wood code selection in (Global) Model Settings

$$C_i := 1.0$$

Incising Factor (always assumed as 1.0 by RISA-3D)

$$C_T := 1.0$$

Buckling Stiffness Factor (always assumed as 1.0 by RISA-3D)

$$C_F := 0.9$$

Size Factor per selected member shape and material  
(per NDS Supplement Table 4A)

$$C_m := 1.0$$

Wet Service Factor per Wood tab of Materials spreadsheet  
(per NDS Supplement Table 4A)

Emin Calculation:

$$COV_E := 0.25$$

Per Table F1 of NDS Appendix F

$$E_{min} := E \cdot \left(1 - 1.645 \cdot COV_E\right) \cdot \left(\frac{1.03}{1.66}\right) = 584493.976 \text{ psi}$$

Per NDS Appendix D Eqn. (D-4)

$$E_{min}' := E_{min} \cdot C_m \cdot C_m \cdot C_m \cdot C_T = 584493.976 \text{ psi}$$

Per NDS Table 4.3.1

Compressive Capacity:

$$\frac{L_z}{d} = 10.868$$

Strong Axis (z-z) slenderness Ratio ( $l_{e1}/d1$ )

$$\frac{L_y}{b} = 40$$

Weak Axis (y-y) slenderness Ratio ( $l_{e2}/d2$ )

$$S := \max\left(\frac{L_z}{d}, \frac{L_y}{b}\right) = 40$$

Maximum Slenderness Ratio

$$F_{cE} := \left(\frac{0.822 \cdot E_{min}'}{S^2}\right) = 300.284 \text{ psi}$$

Per NDS Section 3.7.1

$$F_{c\_star} := F_c \cdot C_D \cdot C_m \cdot C_t \cdot C_F \cdot C_i = 1134 \text{ psi}$$

Per NDS Section 3.7.1

$$c := 0.8$$

Per NDS Section 3.7.1

$$C_P := \left(\frac{1 + \left(\frac{F_{cE}}{F_{c\_star}}\right)}{2 \cdot c}\right) - \sqrt{\left(\left(\frac{1 + \left(\frac{F_{cE}}{F_{c\_star}}\right)}{2 \cdot c}\right)^2 - \left(\frac{\left(\frac{F_{cE}}{F_{c\_star}}\right)}{c}\right)\right)} = 0.2484$$

Per NDS Eqn (3.7-1)

$$E' := E \cdot C_m \cdot C_t \cdot C_i = 1600000 \text{ psi}$$

Per NDS Table 4.3.1

$$F_c' := F_c \cdot C_D \cdot C_m \cdot C_t \cdot C_F \cdot C_i \cdot C_P = 281.6674 \text{ psi}$$

Per NDS Table 4.3.1

Tensile Capacity:

$$F_t' := F_t \cdot C_D \cdot C_m \cdot C_F \cdot C_i = 405 \text{ psi}$$

Per NDS Table 4.3.1

Flexural Capacities:

$$R_B := \sqrt{\frac{L_y \cdot d}{b^2}} = 18.797$$

Per NDS Eqn (3.3-5)

$$F_{bE} := \frac{1.2 \cdot E_{min}'}{R_B^2} = 1985.074 \text{ psi}$$

Per NDS Section 3.3.3

$$F_{b\_star} := F_b \cdot C_D \cdot C_m \cdot C_F \cdot C_i \cdot C_r = 688.5 \text{ psi}$$

Per NDS Section 3.3.3

$$C_{Lz} := \left( \frac{1 + \left( \frac{F_{bE}}{F_{b\_star}} \right)}{1.9} \right) - \sqrt{\left( \left( \frac{1 + \left( \frac{F_{bE}}{F_{b\_star}} \right)}{1.9} \right)^2 - \left( \frac{\left( \frac{F_{bE}}{F_{b\_star}} \right)}{0.95} \right) \right)} = 0.9751 \quad \text{Per NDS Eqn (3.3-6)}$$

Per NDS Section 3.3.3.1

$$C_{Ly} := 1.0$$

$$F_{b1'} := F_b \cdot C_D \cdot C_m \cdot C_{Lz} \cdot C_F \cdot C_i \cdot C_r = 671.3463 \text{ psi} \quad \text{Per NDS Table 4.3.1}$$

$$F_{b2'} := F_b \cdot C_D \cdot C_m \cdot C_{Ly} \cdot C_F \cdot C_{fu} \cdot C_i \cdot C_r = 826.2 \text{ psi} \quad \text{Per NDS Table 4.3.1}$$

#### Shear Capacity:

$$F_v := F_v \cdot C_D \cdot C_m \cdot C_t \cdot C_i = 162 \text{ psi} \quad \text{Per NDS Table 4.3.1}$$

#### Code Check Calculations:

$$F_{cE1} := \frac{0.822 \cdot E_{min'}}{\left( \frac{L_z}{d} \right)^2} = 4067.791 \text{ psi} \quad \text{Per NDS Section 3.9.2}$$

$$F_{cE2} := \frac{0.822 \cdot E_{min'}}{\left( \frac{L_y}{b} \right)^2} = 300.284 \text{ psi} \quad \text{Per NDS Section 3.9.2}$$

#### Max Bending Check:

$$UC_{Max} := \left( \frac{f_a}{F_c'} \right)^2 + \left( \frac{f_{bz}}{F_{b1'} \cdot \left( 1 - \left( \frac{f_a}{F_{cE1}} \right) \right)} \right) + \left( \frac{f_{by}}{F_{b2'} \cdot \left( 1 - \left( \frac{f_a}{F_{cE2}} \right) - \left( \frac{f_{bz}}{F_{bE}} \right)^2 \right)} \right) = 2.429 \quad \text{Per NDS Eqn (3.9-3)}$$

#### Max Shear Check:

$$UC_{Shear} := \frac{f_v}{F_v'} = 1.414 \quad \text{Actual over allowable}$$

### Member M6, Load Combo 3: (DL +LL+Wind)

#### Input & Analysis Values:

Shape Properties (4x4):

$$b := 3.5 \cdot \text{in}$$

$$d := 3.5 \cdot \text{in}$$

$$A := b \cdot d = 12.25 \text{ in}^2$$

$$L_z := 153.675 \cdot \text{in}$$

$$L_y := 153.675 \cdot \text{in}$$

$$S_z := \frac{b \cdot d^2}{6} = 7.146 \text{ in}^3$$

$$S_y := \frac{d \cdot b^2}{6} = 7.146 \text{ in}^3$$

Material Properties (BRC: Southern Pine, Construction):

$$E := 1400000 \cdot \text{psi}$$

$$F_b := 875 \cdot \text{psi}$$

$$F_t := 500 \cdot \text{psi}$$

$$F_v := 175 \cdot \text{psi}$$

$$F_c := 1600 \cdot \text{psi}$$

Design Forces (from the RISA analysis):

$$P := 1388.581 \cdot \text{lbf}$$

Axial force at governing location (0 in)

$$M_z := 0 \cdot \text{ft} \cdot \text{lbf}$$

Strong axis bending moment at governing location (0 in)

$$M_y := 0 \cdot \text{lbf} \cdot \text{ft}$$

Weak axis bending moment at governing location (0 in)

$$V := 14.887 \cdot \text{lbf}$$

Shear force at governing location (153.675 in)

Design Stresses (from the RISA analysis):

$$f_a := \frac{P}{A} = 113.354 \text{ psi}$$

Axial stress per LC3

$$f_{bz} := \frac{M_z}{S_z} = 0 \text{ psi}$$

Strong axis bending stress per LC3

$$f_{by} := \frac{M_y}{S_y} = 0 \text{ psi}$$

Weak axis bending stress per LC3

$$f_v := \frac{3 \cdot V}{2 \cdot A} = 1.823 \text{ psi}$$

Shear Stress per LC3

#### Design Calculations:

Load Factors (per input variables):

$$C_D := 1.6$$

Load Duration Factor per Design tab of Load Combinations spreadsheet

$$C_r := 1.0$$

Repetitive Member Factor per Wood tab of Members spreadsheet

$$C_{fu} := 1.0$$

Flat Use Factor per member orientation relative to loading

$$C_t := 1.0$$

Temperature Factor per Wood code selection in (Global) Model Settings

$$C_i := 1.0$$

Incising Factor (always assumed as 1.0 by RISA-3D)

$$C_T := 1.0$$

Buckling Stiffness Factor (always assumed as 1.0 by RISA-3D)

$$C_F := 1.0$$

Size Factor per selected member shape and material  
(per NDS Supplement Table 4A)

$$C_m := 1.0$$

Wet Service Factor per Wood tab of Materials spreadsheet  
(per NDS Supplement Table 4A)



---

Emin Calculation:

$$COV_E := 0.25$$

Per Table F1 of NDS Appendix F

$$E_{min} := E \cdot \left(1 - 1.645 \cdot COV_E\right) \cdot \left(\frac{1.03}{1.66}\right) = 511432.229 \text{ psi}$$

Per NDS Appendix D Eqn. (D-4)

$$E_{min}' := E_{min} \cdot C_m \cdot C_m \cdot C_m \cdot C_T = 511432.229 \text{ psi}$$

Per NDS Table 4.3.1

Compressive Capacity:

$$\frac{L_z}{d} = 43.9071$$

Strong Axis (z-z) slenderness Ratio ( $l_e1/d1$ )

$$\frac{L_y}{b} = 43.9071$$

Weak Axis (y-y) slenderness Ratio ( $l_e2/d2$ )

$$S := \max\left(\frac{L_z}{d}, \frac{L_y}{b}\right) = 43.9071$$

Maximum Slenderness Ratio

$$F_{cE} := \left(\frac{0.822 \cdot E_{min}'}{S^2}\right) = 218.067 \text{ psi}$$

Per NDS Section 3.7.1

$$F_{c_{star}} := F_c \cdot C_D \cdot C_m \cdot C_t \cdot C_F \cdot C_i = 2560 \text{ psi}$$

Per NDS Section 3.7.1

$$c := 0.8$$

Per NDS Section 3.7.1

$$C_P := \left(\frac{1 + \left(\frac{F_{cE}}{F_{c_{star}}}\right)}{2 \cdot c}\right) - \sqrt{\left(\left(\frac{1 + \left(\frac{F_{cE}}{F_{c_{star}}}\right)}{2 \cdot c}\right)^2 - \left(\frac{\left(\frac{F_{cE}}{F_{c_{star}}}\right)}{c}\right)\right)} = 0.0837$$

Per NDS Eqn (3.7-1)

$$E' := E \cdot C_m \cdot C_t \cdot C_i = 1400000 \text{ psi}$$

Per NDS Table 4.3.1

$$F_c' := F_c \cdot C_D \cdot C_m \cdot C_t \cdot C_F \cdot C_i \cdot C_P = 214.1566 \text{ psi}$$

Per NDS Table 4.3.1

Tensile Capacity:

$$F_t' := F_t \cdot C_D \cdot C_m \cdot C_F \cdot C_i = 800 \text{ psi}$$

Per NDS Table 4.3.1

Flexural Capacities:

$$R_B := \sqrt{\frac{L_y \cdot d}{b^2}} = 6.6262$$

Per NDS Eqn (3.3-5)

$$F_{bE} := \frac{1.2 \cdot E_{min}'}{R_B^2} = 13977.65 \text{ psi}$$

Per NDS Section 3.3.3

$$F_{b_{star}} := F_b \cdot C_D \cdot C_m \cdot C_F \cdot C_i \cdot C_r = 1400 \text{ psi}$$

Per NDS Section 3.3.3

$$C_{Lz} := 1.0 \quad \text{Per NDS Section 3.3.3.1}$$

$$C_{Ly} := 1.0 \quad \text{Per NDS Section 3.3.3.1}$$

$$F_{b1'} := F_b \cdot C_D \cdot C_m \cdot C_{Lz} \cdot C_F \cdot C_i \cdot C_r = 1400 \text{ psi} \quad \text{Per NDS Table 4.3.1}$$

$$F_{b2'} := F_b \cdot C_D \cdot C_m \cdot C_{Ly} \cdot C_F \cdot C_{fu} \cdot C_i \cdot C_r = 1400 \text{ psi} \quad \text{Per NDS Table 4.3.1}$$

Shear Capacity:

$$F_{v'} := F_v \cdot C_D \cdot C_m \cdot C_t \cdot C_i = 280 \text{ psi} \quad \text{Per NDS Table 4.3.1}$$

Code Check Calculations:

Max Bending Check:

$$UC_{Max} := \left( \frac{f_a}{F_c'} \right) = 0.529 \quad \text{Per NDS Section (3.6.3)}$$

Max Shear Check:

$$UC_{Shear} := \frac{f_v}{F_{v'}} = 0.007 \quad \text{Actual over allowable}$$

\*Note: For some members the limitations in section 3.6.3 control over any of the equations. This is because in the Compression-Bending Interaction equation (Eqn. 3.9-3), if the bending goes to zero, the equation will automatically square the compression portion, lowering it from what we know to be the actual capacity ( $f_c/F_c'$  vs.  $(f_c/F_c')^2$ ). This section allows us to use the compression portion without squaring it to know the true capacity of the compression-only member.

## Comparison

NDS 2015 Wood Bending Check Comparisons				
Member	Load Combo	RISA-3D	Hand Calc	% Difference
M1	3	0.313	0.314	0.32
M2	2	0.254	0.254	0.00
M3	3	3.047	3.047	0.00
M5	1	2.429	2.429	0.00
M6	3	0.529	0.529	0.00

Table 10.1 – Bending Unity Check Comparison

## Conclusion

As seen in the chart above, the results match very closely. The cause for any slight differences can be attributed to numerical round off.

# Verification Problem 11

---

## Problem Statement

This problem is used to test the tapered WF sections. A typical single bay with a sloped roof (see Fig. 11.1) will be analyzed using tapered WF sections for the columns and beams. Loading will consist of vertical member projected loads, lateral member distributed loads, and member point loads. Gravity self-weight will also be applied.

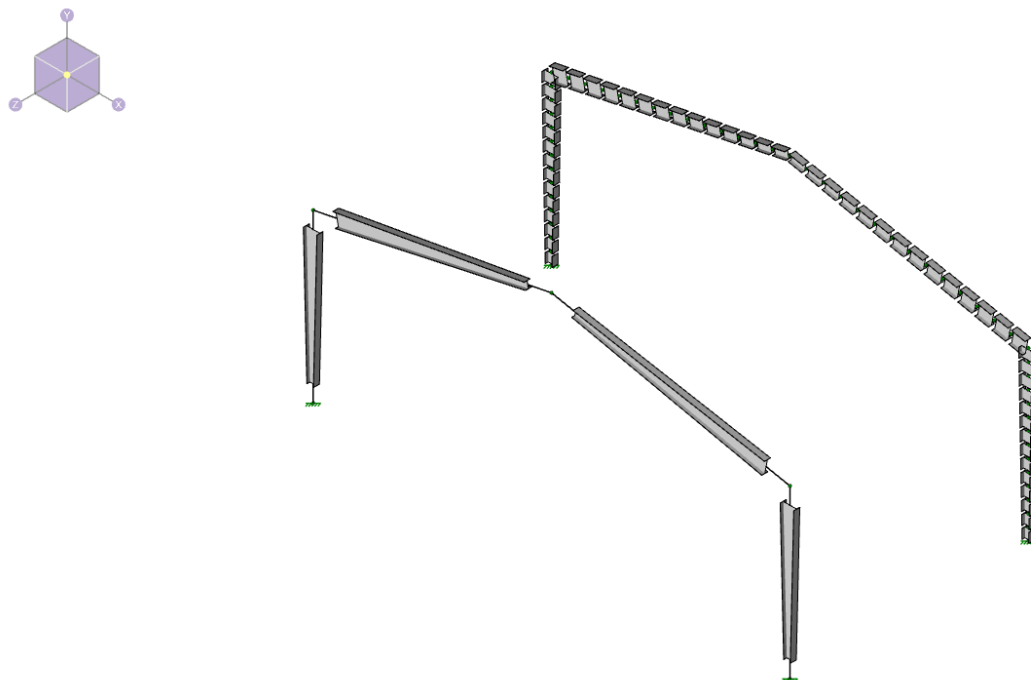


Figure 11.1- Model Sketch of Frames

## Validation Method

The frame analyzed with tapered WF sections will be compared to a similar frame, which is modeled with 14 piecewise prismatic sections for each tapered WF member in the original frame (see Fig. 11.1). Since each tapered WF member is modeled internally as a 14 piecewise prismatic “member,” the results should match very closely. Selected joint deflections, reactions, and member section forces will be compared (see Tables 11.1-11.3). The ASD code checks on the tapered WF sections (for member properties see Table 11.4) will be compared to hand calculations using the *AISC 360-22 (16<sup>th</sup> Ed.) ASD Steel Code* and the *AISC Design Guide #25: Frame Design Using Web-Tapered Members*.

## Comparison

Comparison of Joint Deflections – Load Combination 1					
Tapered WF Frame			Equivalent "Piecewise" Frame		
Node	Direction	Deflection (in)	Node	Direction	Deflection (in)
N2	X	-0.877	N7	X	-0.877
N3	Y	-3.002	N8	Y	-3.002
N4	X	0.290	N9	X	0.290

Table 11.1 – Joint Deflections

The joint deflections were checked at the top left corner, peak, and top right corner, respectively. As is seen in the chart above, the results match exactly.

Comparison of Base Reactions – Load Combination 1							
Tapered WF Frame				Equivalent "Piecewise" Frame			
Node	X (k)	Y (k)	MZ (k-ft)	Node	X (k)	Y (k)	MZ (k-ft)
N1	5.659	18.533	0	N6	5.659	18.533	0
N5	-10.859	17.091	41.749	N10	-10.859	17.091	41.750

Table 11.2 – Base Reactions

The reactions were checked at the two base nodes. As seen above, the results match almost exactly.

Comparison of Member Section Forces – Load Combination 1							
Tapered WF Frame				Equivalent "Piecewise" Frame			
Member	Section Cut Location	Local Direction	Value (k, or k-ft)	Member	Section Cut Location	Local Direction	Value (k, or k-ft)
M1	5	Mz	108.629	M18	5	Mz	108.631
M1	1	x	18.533	M5	1	x	18.533
M2	5	y	-15.916	M32	5	y	-15.914
M2	5	Mz	108.628	M32	5	Mz	108.631
M2	1	Mz	-30.972	M19	1	Mz	-30.97
M3	1	Mz	-30.972	M47	1	Mz	-30.97
M3	5	Mz	99.779	M60	5	Mz	99.781
M3	5	y	-14.501	M60	5	y	-14.499
M4	5	Mz	-99.78	M46	5	Mz	-99.781
M4	1	x	17.091	M33	1	x	17.091

Table 11.3 – Member Forces

The section forces were checked at the base of the columns, at the corner joints, and at the peak. As can be seen in the chart above, the results match almost exactly.

### ***Tapered Section Properties***

Tapered WF Properties		
	Taper Start	Taper End
Total Depth (in)	7	14
Web Thickness (in)	0.25	0.25
Flange Width (in)	6	6
Flange Thickness (in)	0.375	0.375

Table 11.4 – Section Properties

### ***AISC 15<sup>th</sup> Ed. (and AISC Design Guide 25) ASD Code Check for M2, Load Combination 2:***

#### ***Cross Sectional Properties:***

$I_{zm} := 134.692 \cdot \text{in}^4$	Moment of Inertia at midpoint (Strong Axis)
$I_{ym} := 13.513 \cdot \text{in}^4$	Moment of Inertia at midpoint (Weak Axis)
$A_m := 6.938 \cdot \text{in}^2$	Area at midpoint
$r_{zm} := \sqrt{\frac{I_{zm}}{A_m}} = 4.406 \text{ in}$	Radius of Gyration at midpoint (Strong Axis)
$r_{ym} := \sqrt{\frac{I_{ym}}{A_m}} = 1.396 \text{ in}$	Radius of Gyration at midpoint (Weak Axis)
$J_m := 0.259 \cdot \text{in}^4$	Torsional J at midpoint
$C_{wm} := 346.32 \cdot \text{in}^6$	Warping Constant at midpoint
$A_{ee} := 7.4585 \cdot \text{in}^2$	Effective area at ending end
$S_{ze} := 36.766 \cdot \text{in}^3$	Elastic Section Modulus at ending end (Strong Axis)
$S_{ye} := 4.506 \cdot \text{in}^3$	Elastic Section Modulus at ending end (Weak Axis)
$Z_{ze} := 41.629 \cdot \text{in}^3$	Plastic Section Modulus at ending end (Strong Axis)
$Z_{ye} := 6.957 \cdot \text{in}^3$	Plastic Section Modulus at ending end (Weak Axis)
$\Omega := 1.67$	

#### ***Loading (Per RISA Analysis):***

Governing location: 244.75 in  
 $P := 13.3233 \cdot \text{kip}$   
 $M_{rz} := 112.049 \cdot \text{kip} \cdot \text{ft}$   
 $M_{ry} := 0 \cdot \text{kip} \cdot \text{ft}$

#### ***Unbraced Lengths:***

$K := 1.0$   
 $L_y := 12 \cdot \text{in}$   
 $L_z := 244.7529 \cdot \text{in}$   
 $L_{comp} := 12 \cdot \text{in}$

#### ***Material Properties:***

$F_y := 50 \cdot \text{ksi}$   
 $E := 29000 \cdot \text{ksi}$   
 $G := 11154 \cdot \text{ksi}$

### Axial Capacity Calculations:

$$P_{et} := \left( \left( \left( \frac{\pi^2 \cdot E \cdot C_{wm}}{(K \cdot L_z)^2} \right) + G \cdot J_m \right) \cdot \left( \frac{1}{r_{ym}^2 + r_{zm}^2} \right) \right) = (2.127 \cdot 10^5) \text{ lbf} \quad (\text{Per DG Eqn 5.3-12})$$

$$F_e := \frac{P_{et}}{A_{ee}} = 28517.984 \text{ psi}$$

$$\frac{F_y}{F_e} = 1.753 < 2.25$$

$$F_{cr} := \left( 0.658^{\left( \frac{F_y}{F_e} \right)} \right) \cdot F_y = 24.003 \text{ ksi} \quad (\text{Per Eqn E7-2})$$

$$P_n := F_{cr} \cdot A_{ee} = 179.028 \text{ kip} \quad (\text{Per Eqn E7-1})$$

$$P_c := \frac{P_n}{\Omega} = 107.202 \text{ kip}$$

### Flexural Capacity Calculations (Strong Axis):

$$h_c := 6.25 \cdot \text{in}$$

$$t_w := 0.25 \cdot \text{in}$$

$$\frac{h_c}{t_w} = 25 < \lambda_{pw} := 3.76 \cdot \sqrt{\frac{E}{F_y}} = 90.553$$

$$M_p := \min((F_y \cdot Z_{xe}), (1.6 \cdot F_y \cdot S_{xe})) = 173.454 \text{ kip} \cdot \text{ft}$$

$$M_{yc} := F_y \cdot S_{xe} = 153.192 \text{ kip} \cdot \text{ft}$$

$$\text{Therefore, } R_{pc} := \frac{M_p}{M_{yc}} = 1.132 \quad (\text{Per DG Eqn 5.4-4})$$

$$\frac{h_c}{t_w} = 25 < \lambda_{rw} := 5.7 \cdot \sqrt{\frac{E}{F_y}} = 137.274$$

$$\text{Therefore, } R_{pg} := 1.0$$

$$M_{nz} := R_{pc} \cdot R_{pg} \cdot M_{yc} = 173.454 \text{ kip} \cdot \text{ft} \quad (\text{Per DG Eqn 5.4-8})$$

$$M_{cz} := \frac{M_{nz}}{\Omega} = 103.865 \text{ kip} \cdot \text{ft}$$

### Flexural Capacity Calculations (Weak Axis):

$$M_{ny} := \min((F_y \cdot Z_{ye}), (1.6 \cdot F_y \cdot S_{ye})) = 28.988 \text{ kip} \cdot \text{ft} \quad (\text{Per Eqn F6-1})$$

$$M_{cy} := \frac{M_{ny}}{\Omega} = 17.358 \text{ kip} \cdot \text{ft}$$

Max Bending Check:

$$\frac{P}{2 \cdot P_c} = 0.062 < 0.2$$

$$\left( \frac{P}{2 \cdot P_c} \right) + \left( \frac{Mr_z}{Mc_z} \right) + \left( \frac{Mr_y}{Mc_y} \right) = 1.141 \quad \text{(Per Eqn H1-1b)}$$

## Conclusion

As seen above, the results match the RISA-3D result within a reasonable amount of error.



# Verification Problem 12

---

## Problem Description

This problem represents a 10 story moment resistant steel frame. This model tests the first- and second- order lateral displacements (see Figure 12.1) by using several different methods both in RISA-3D and by hand. These methods are based on satisfying the new P-Delta design requirements found in current design codes. The hand verification of this problem is similar to that given in The Seismic Design Handbook by Farzad Naeim (Example 7-1).

A model was built per the description given in the text. The beams and columns were entered as the given wide flange sections shown in Figure 12.3. The applied loads were entered as those given in Figure 12.2.

The lateral displacements of each level were calculated using several different methods, first by those presented in the example and then in RISA-3D. These values were then compared to one another in order to examine the effect of P-Delta on the lateral displacement of frames.

### *P-Delta Displacements*

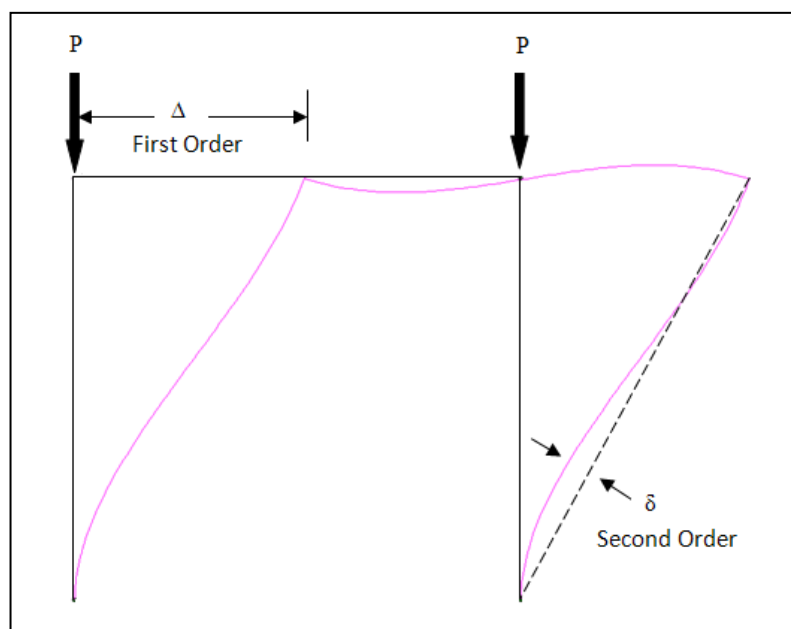
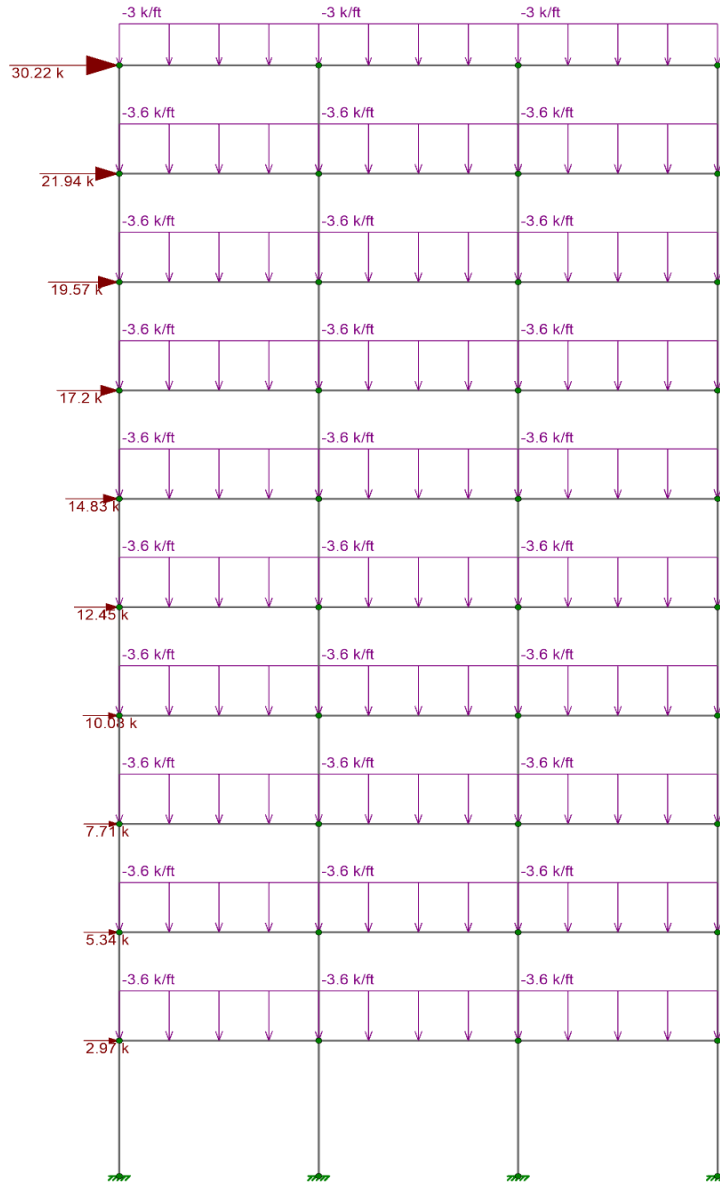
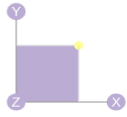


Figure 12.1 – P-Delta Concept

A model was built per the description given in the example.

Lateral Loads	=	Varies by level (see Figure 12.2)
Gravity Load- Floor	=	120 psf
Gravity Load – Roof	=	100 psf
Frame Tributary Width	=	30 ft
Story Height	=	Varies by level (see Figure 12.3)



Loads: LC 1, Basic

Figure 12.2- Moment Frame Elevation with Applied Loads Shown



Figure 12.3 - Moment Frame Elevation with Member Sizes and Dimensions Shown

## Validation Method

### *SDH Methods*

The Seismic Design Handbook utilizes two methods for analyzing the second order P-delta effects. The first is an iterative process where an analytical model is first used to compute the first order displacements from the applied loads. These displacements are then re-applied to the model as secondary shears giving the user a modified set of displacements. This process is repeated until a reasonable convergence of data produces the final lateral displacement. See Table 12.2 for a comparison of these deflections versus those of the RISA-3D P-Delta feature, below.

The second method, the Non-Iterative P-delta Method, is a hand calculated simplification of the iterative method. Using the assumption that story drift at any level is proportional only to the applied story shear at that level, the first order deflections are calculated using an applied lateral load and then multiplied by a magnification factor to account for the second order P-delta effects.

**Note:** Because the example calculation does not account for axial shortening of the columns, the elastic analysis in their methods differs by up to 2% from that of other methods outlined in this example.

### *SDH Comparison*

The graph (Figure 12.4) below shows the minimal difference between the SDH Methods.

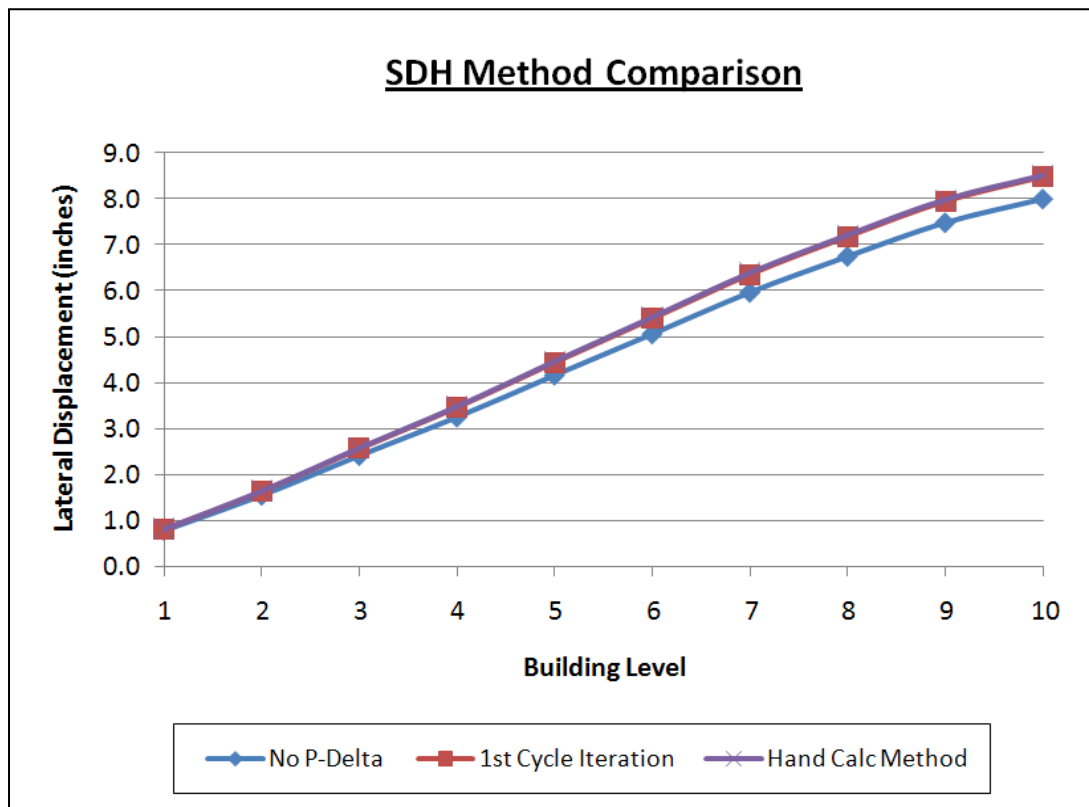


Figure 12.4 - Comparison of Deflections from each SDH Method

Deflection Results Comparison (inches)			
Level	SDH Modified Force Method	RISA-3D* with P-Delta	% Difference
10	8.6706	8.6853	0.169
9	8.1308	8.1450	0.174
8	7.3534	7.3668	0.182
7	6.5166	6.5291	0.192
6	5.5394	5.5504	0.198
5	4.5622	4.5715	0.204
4	3.5614	3.5688	0.208
3	2.6412	2.6467	0.208
2	1.6856	1.6890	0.202
1	0.8393	0.8410	0.202

Table 12.1– SDH Deflection Comparison

\*Results will differ in RISA-2D due to lack of rigid diaphragms

The program results match within a reasonable round off error.

### ***RISA-3D Methods***

In RISA-3D, P-Δ effects are accounted for whenever the user requests it in the Load Combinations spreadsheet. But because RISA-3D second order analysis is based entirely on nodal deflections, the effect of P-δ is not directly accounted for. Therefore, the user must place additional nodes along the column length to account for the P-δ effects. This can be done with any number of additional nodes; with more nodes, the more accurate the solution. Please see Figure 12.4 below for a comparison of these effects on the solution. The RISA-3D (with P-Δ & P-δ) values in Table 12.3 are obtained using 2 intermediate nodes on each column.

The hand calculation method used to verify the program results is the Non-Iterative Method from the Seismic Design Handbook. In this method, the first order lateral displacements are used to find  $\Theta$ , the Stability Index. The amplified shear values are then found by multiplying the first order lateral displacements by  $1/(1-\Theta)$ , see Table 12.2 below.

Non-Iterative Method Amplified Shears			
Level	Applied Story Shear (k)	Stability Index ( $\Theta$ )	Amplified Shear (k)
10	30.22	0.02	30.89
9	21.94	0.05	23.12
8	19.57	0.06	20.84
7	17.20	0.08	18.70
6	14.83	0.09	16.34
5	12.45	0.11	14.03
4	10.08	0.13	11.55
3	7.71	0.17	9.32
2	5.34	0.22	6.85
1	2.97	0.32	4.35

Table 12.2 - Direct Hand Method  $\Theta$  Values and Amplified Shears

### RISA-3D Comparison

The graph (Figure 12.4) below shows the minimal difference between the RISA Methods.

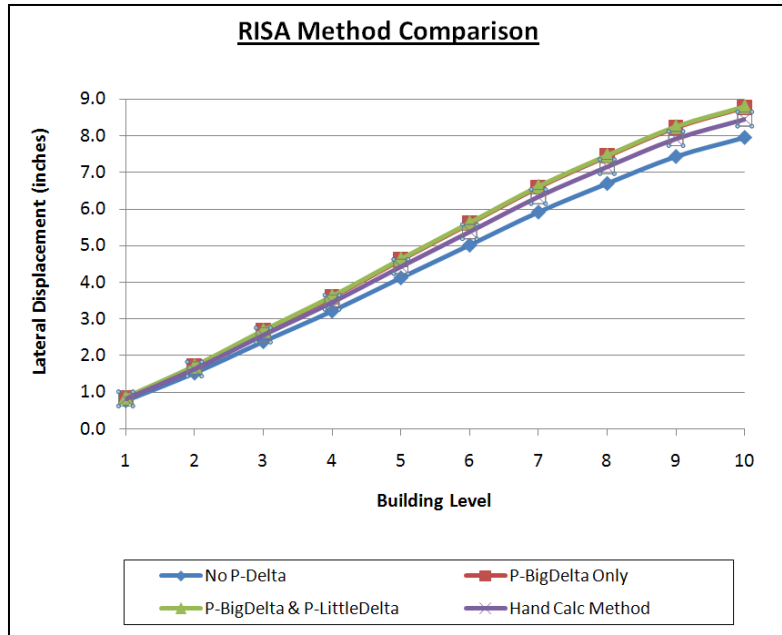


Figure 12.4 - Comparison of Deflections from Each RISA Method

Deflection Results Comparison (inches)				
Level	Non-Iterative Method	RISA-3D with P-Δ	RISA-3D with P-Δ & P-δ	% Increase for P-δ
10	8.6686	8.6853	8.6955	0.117
9	8.1299	8.1450	8.1551	0.124
8	7.3560	7.3668	7.3765	0.132
7	6.5240	6.5291	6.5383	0.141
6	5.5547	5.5504	5.5587	0.150
5	4.5843	4.5715	4.5790	0.164
4	3.5891	3.5688	3.5754	0.185
3	2.6699	2.6467	2.6526	0.223
2	1.7131	1.6890	1.6937	0.278
1	0.8581	0.8410	0.8438	0.333

Table 12.3 – Non-Iterative Method Deflection Comparison

### Conclusion

The program results match the textbook example within a reasonable round off error.

# Verification Problem 13

## Problem Statement

This model is a planar frame structure consisting of seven simply-supported W14x68 beams at a 30 degree incline to the vertical Y-axis (see Fig. 13.1 below). A 0.1ksf area load is applied to the frame in the Z direction. Some of the beams are rotated about their local x-axis as noted below. Here we test distribution of member area loads for the Projected Area Only option, using both global and projected directions.

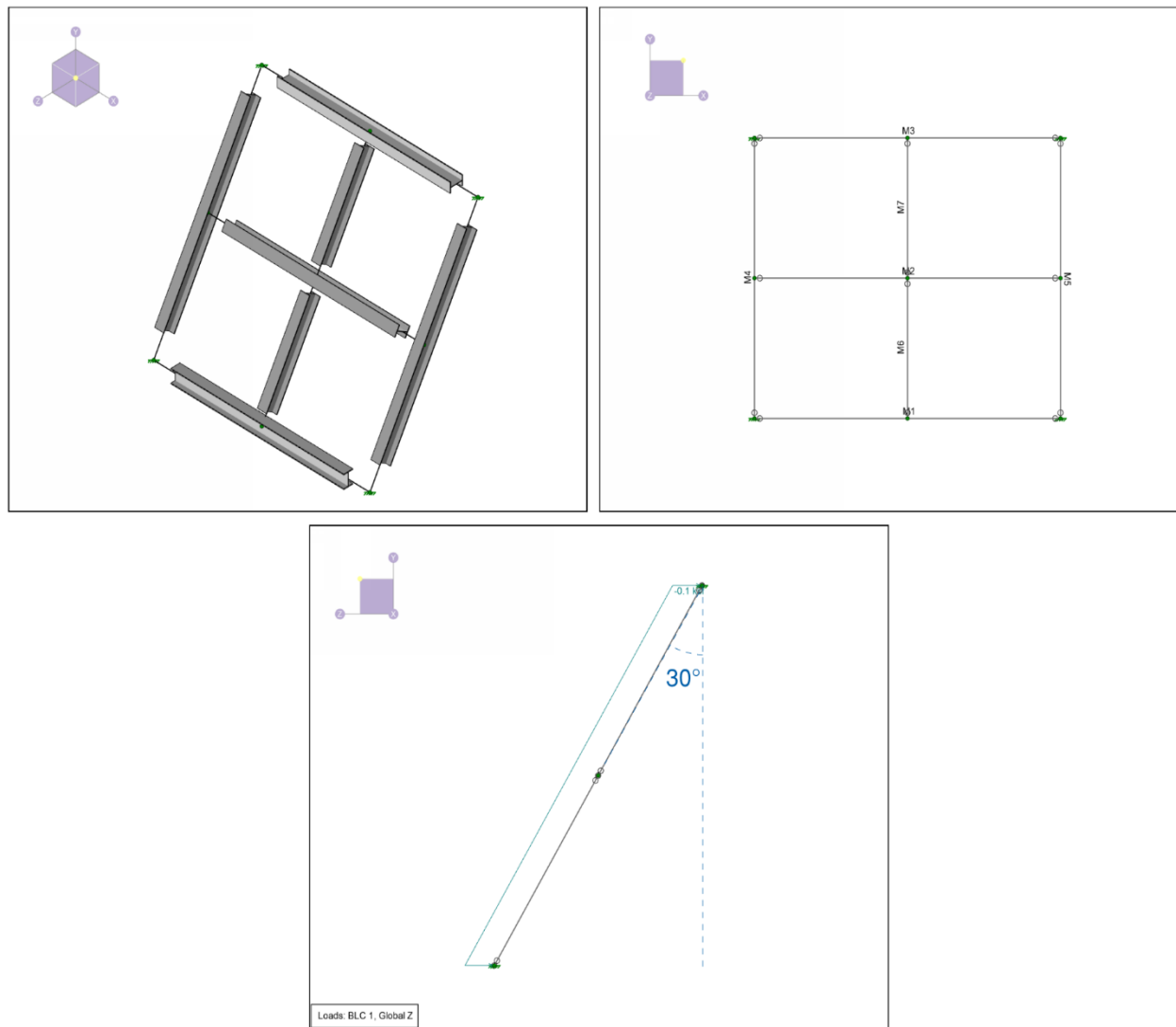


Figure 13.1- Model Views

## Validation Method

Envelope dimensions of the projected sections are used to calculate equivalent uniform member distributed loads. The projected section depth and width:

$$d_{projected} = d * \cos\phi$$

$$b_{f_{projected}} = b_f * \sin\phi$$

$$Total\ Projected\ Width = d_{projected} + b_{f_{projected}}$$

Equivalent uniform member distributed loads can then be calculated for both the Global Z and Projected Z directions:

$$\omega_{z\ Global\ Loads} = \frac{d_{projected}}{\cos(\theta)} * \rho$$

$$\omega_{z\ Projected\ Loads} = d_{projected} * \rho$$

Where  $\theta$  = vertical angle [deg.]

$\phi$  = local axis rotation angle [deg.]

$d$  = total section depth [in.]

$b_f$  = total section width [in.]

$d_{projected}$  = projected section depth [in.]

$\omega$  = equivalent uniform member distributed load [k/ft]

$\rho$  = uniform member area load [ksf]

Z Direction Global Loads								
Member	Shape	d (in)	bf (in)	$\theta$ (deg.)	$\phi$ (deg.)	$\rho$ (ksf)	Tot. Projected Width (in)	$\omega Z$ (klf)
M1	W14X68	14	10	30	0	0.1	14.00	0.135
M2	W14X68	14	10	30	60	0.1	15.66	0.151
M3	W14X68	14	10	30	90	0.1	10.00	0.096

Table 13.1 – Global Direction Hand Calculations

Z Direction Projected Loads							
Member	Shape	d (in)	bf (in)	$\phi$ (deg.)	$\rho$ (ksf)	Tot. Projected Width (in)	$\omega Z$ (klf)
M1	W14X68	14	10	0	0.1	14.00	0.117
M2	W14X68	14	10	60	0.1	15.66	0.131
M3	W14X68	14	10	90	0.1	10.00	0.083

Table 13.2 – Projected Direction Hand Calculations



## Comparison

Equivalent Uniform Member Distributed Loads, $\omega Z$						
Member	Global Z (k/ft)			Projected Z (k/ft)		
	Theoretical	RISA-3D	%Diff.	Theoretical	RISA-3D	%Diff.
M1	0.135	0.135	0.000	0.117	0.117	0.000
M2	0.151	0.151	0.000	0.131	0.131	0.000
M3	0.096	0.096	0.000	0.083	0.083	0.000

Table 13.3 – Load Calculation Comparison

## Conclusion

As seen in Table 13.3 above, the results match exactly.

# Verification Problem 14

---

## Problem Statement

This model is a comparison of a concrete beam cantilever created with solids elements versus one modeled with the concrete beam element. Both are loaded with vertical point loads at the free end.

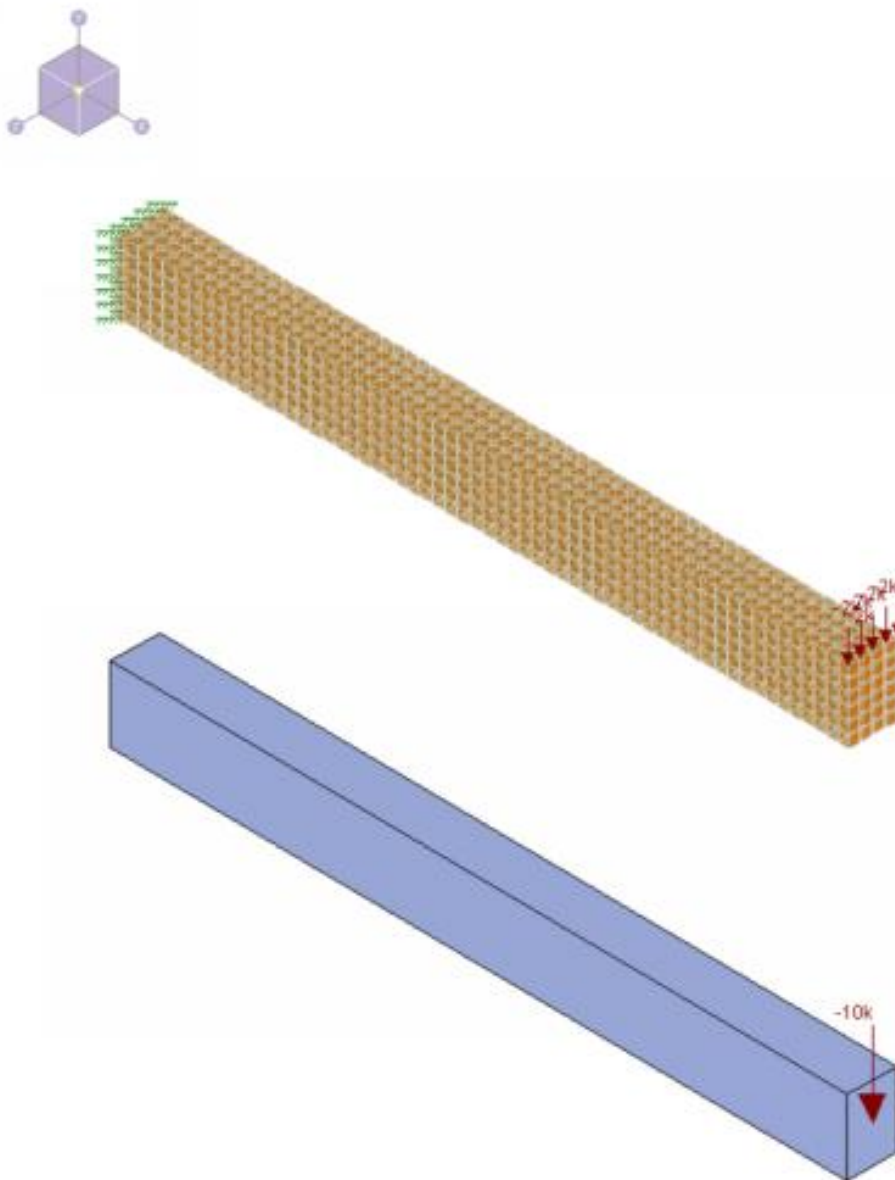


Figure 14.1 – Model View

## Validation Method

The deflections at the tip of each cantilever are compared to the values obtained by hand calculations. Deflection at the tip of a cantilever beam is calculated as follows:

$$\Delta_{bending} = \frac{P * L^3}{3 * E * I}$$

Where,

P = 10 kips

L = 10 ft = 120 in

E = 3644 ksi (Conc4NW material)

I = 1152 in<sup>4</sup>

Therefore, per our hand calculation,  $\Delta_{bending} = 1.372 \text{ in}$ .

## Comparison

For this model:

Beam Deflection Comparison			
Element	Node	RISA-3D Bending Deflection (in)	% Difference
Solids	N1115	-1.361	0.80
Beam	N2137	-1.372	0.00

Table 14.1 – Load Calculation Comparison

## Conclusion

As seen in Table 14.1 above, the results are within a reasonable difference from the hand calculations.

# Verification Problem 15

---

## Problem Statement

This model is a collection of members that verifies the AISC 360-22 specification for tension members from the AISC Design Examples 16<sup>th</sup> edition. Each of these is using the ASD design parameters and uses parameters from the individual problems.

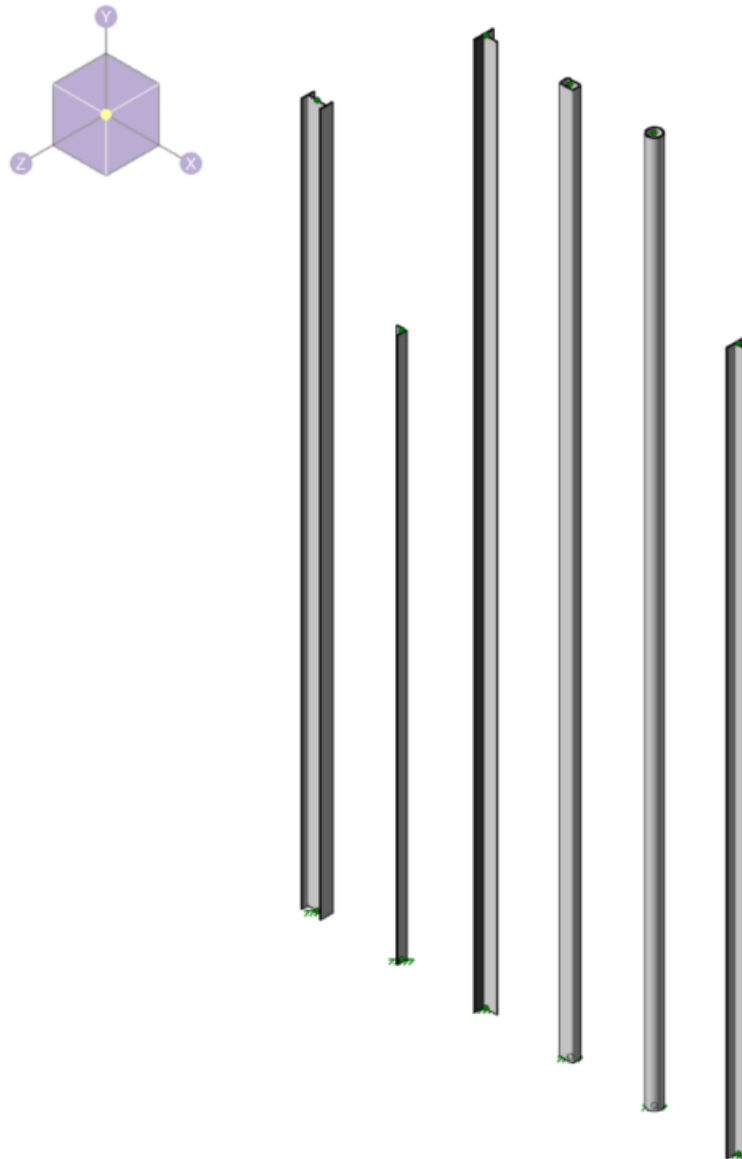


Figure 15.1 – Model View

## Validation Method

In this example we are simply checking the tensile yield limit state. RISA does not know specific bolt hole locations, therefore it does not check tensile rupture limit states.

## Comparison

For this model:

Example	Shape	RISA Value (kips)	AISC Value (kips)	% Difference
D.1	W8X21	184.431	184	0.23
D.2	L4X4X1/2	80.838	80.8	0.05
D.3	WT6X20	174.85	175	0.09
D.4	HSS6X4X3/8	185.03	185	0.16
D.5	HSS6x0.500	222.838	223	0.07
D.6	2L4X4X1/2 (1/2" Gap)	161.677	162	0.20

Table 15.1 – Tensile Yield Capacity comparison

## Comparison

As seen in Table 15.1 above, the results are within a reasonable difference from the AISC hand calculations.

# Verification Problem 16

---

## Problem Statement

This model is a collection of members that verifies the AISC 360-22 specification for compression members from the AISC Design Examples 16<sup>th</sup> edition. Each of these is using the ASD design parameters and uses parameters from the individual problems.

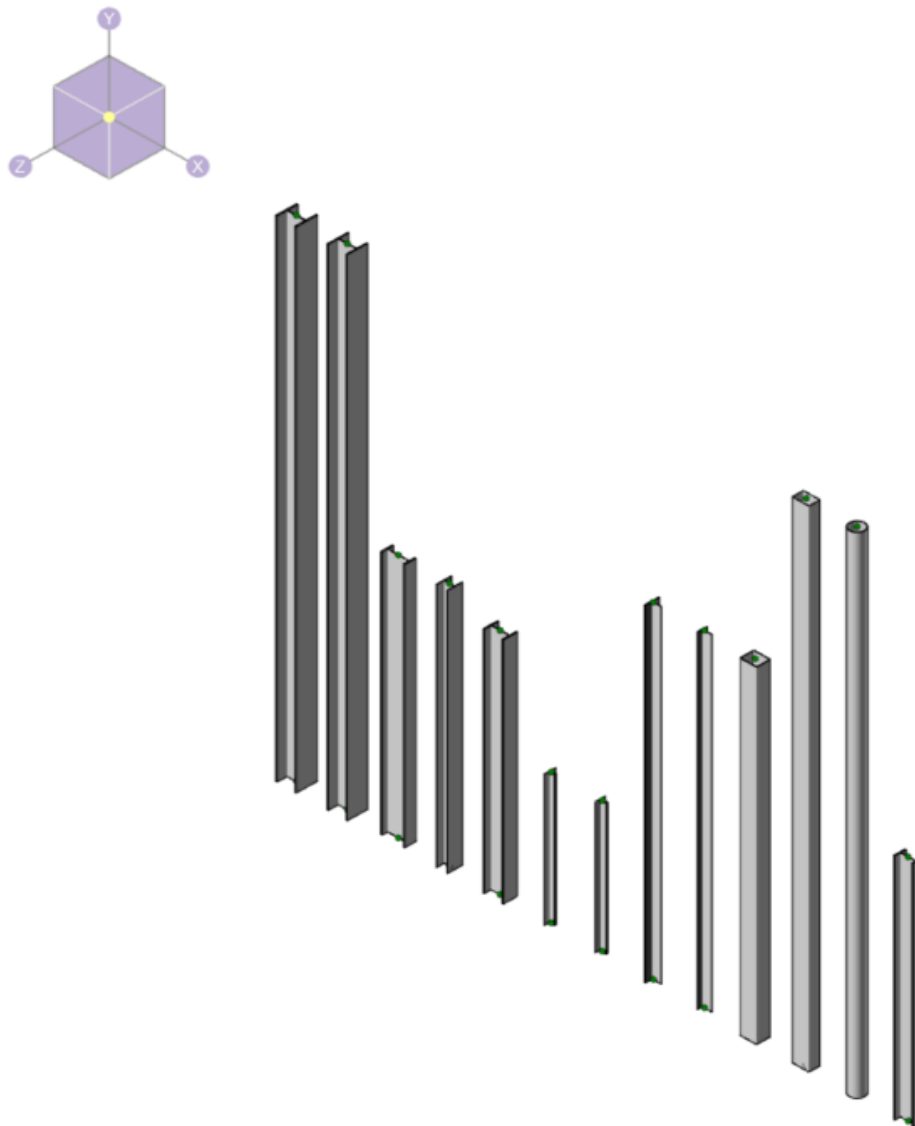


Figure 16.1 – Model View

## Validation Method

In this example we are checking the compression capacity of members for all AISC limit states. In many cases there is a “Table Solution” and a “Calculation Solution”. In each of these cases we are listing the “Calculation Solution”.

## Comparison

This section is the tabular comparison of the RISA Program answers and the summary from the detailed validation results.

Example	Shape	RISA Value (kips)	AISC Value (kips)	% Difference
E.1A	W14X132	593.89	594	0.02
E.1B	W14X90	600.70	601	0.05
E.2	WF (Slender Web)	331.29	332	0.21
E.3	WF (Slender Flange)	211.22	211	0.10
E.4A	W14X82 (Col B-C)*	625.80	626	0.03
E.5	LL4X3.5X3/8 (3/4" Gap)	84.47	85.0	0.63
E.6	LL3X5X1/4 (3/4" Gap)	45.61	45.4	0.46
E.7	WT7X34	85.07	85.0	0.08
E.8	WT7X15	24.30	24.4	0.43
E.9	HSS12X10X3/8	369.46	370	0.15
E.10	HSS12X8X3/16	100.68	101	0.32
E.11	Pipe 10 Std.	145.43	148	1.74**
E.12	Built-Up Unequal Flange	184.50	186	0.81

Table 16.1 – Compression Capacity comparison

\*Note that the K for this shape was set to 1.568. The example defines K = 1.5. However, the example yields a KL = 8.61', but a conservative 9' is used. By taking K in RISA-3D =  $1.5 \cdot (9/8.61) = 1.568$  we can approach the hand calculated value.

\*\*Note that Table 1-14 in the AISC 360-22 reports  $r = 3.68$ " for a Pipe 10 Std. RISA-3D internally calculates  $r$  as  $\sqrt{I/A} = \sqrt{(151 \text{ in}^4 / 11.5 \text{ in}^2)} = 3.62$ ".

## Conclusion

As seen in Table 16.1 above, the results are within a reasonable difference from the AISC hand calculations.

# Verification Problem 17

---

## Problem Statement

This model is a collection of members that verifies the AISC 360-22 specification for flexural members from the AISC Design Examples 16<sup>th</sup> edition. Each of these is using the ASD design parameters and is built with the exact specifications from the example problems.

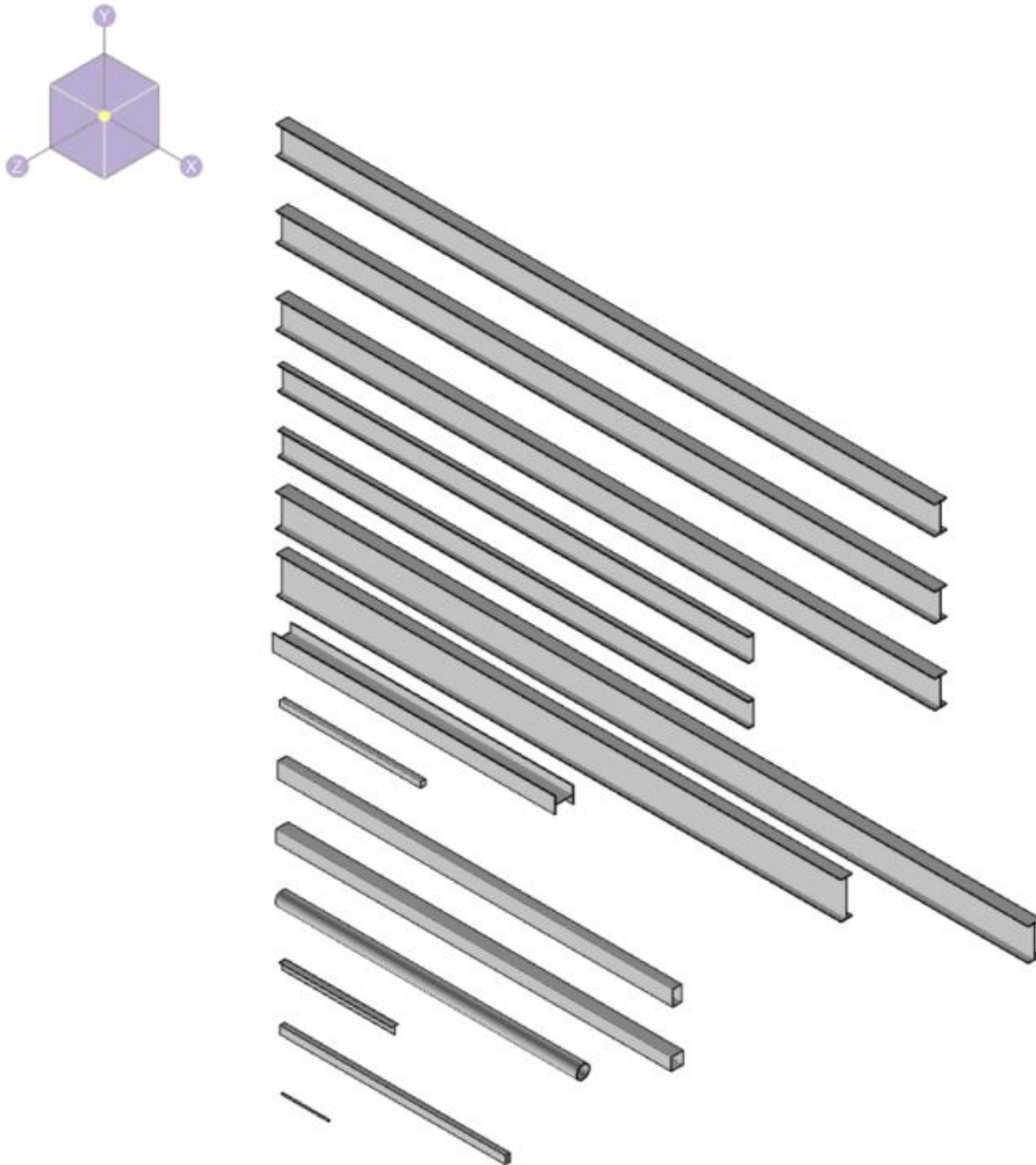


Figure 17.1 – Model View



## Validation Method

In this example we are checking the flexural strength of members subject to simple bending about one principal axis as well as member deflections in some of the members.

## Comparison

Example	LC	Capacity (k*ft)	RISA Value	AISC Value	% Difference
F.1-1	1	$M_{nz}/\Omega$	251.996	252	0.00
F.1-2	1	$M_{nz}/\Omega$	201.268	201	0.13
F.1-3	1	$M_{nz}/\Omega$	191.206	192	0.41
F.2-1	1	$M_{nz}/\Omega$	91.257	91.3	0.05
F.2-2	1	$M_{nz}/\Omega$	87.148	87	0.17
F.3A	1	$M_{nz}/\Omega$	264.775	265	0.08
F.4	1	$M_{nz}/\Omega$	334.331	334	0.10
F.5	1	$M_{ny}/\Omega$	81.088	81.4	0.38
F.6	1	$M_{nz}/\Omega$	4.796	4.79	0.13
F.7	1	$M_{nz}/\Omega$	39.79	39.7	0.23
F.8	1	$M_{nz}/\Omega$	30.864	30.8	0.21
F.9	1	$M_{nz}/\Omega$	54.142	54.1	0.08
F.10	1	$M_{nz}/\Omega$	4.851	4.87	0.39
F.12	1	$M_{nz}/\Omega$	33.683	33.8	0.35
F.13	1	$M_{nz}/\Omega$	0.282	0.283	0.35

Table 17.1 – Flexural Capacity Comparison

Example	Deflection (in)	LC	RISA Value	AISC Value	% Difference
F.2-1	Live Load Deflection	2	0.664	0.664	0.00
F.3	Total Deflection	1	2.644	2.66	0.60
F.8	Live Load Deflection	2	1.04	1.04	0.00

Table 17.2 – Member Deflection Comparison

## Conclusion

As seen in the tables above, the results are within a reasonable difference from the AISC hand calculations.

# Verification Problem 18

---

## Problem Statement

This model is a collection of members that verifies the AISC 360-22 specification for shear members from the AISC Design Examples 16<sup>th</sup> edition. Each of these is using the ASD design parameters and is built with the exact specifications from the example problems.

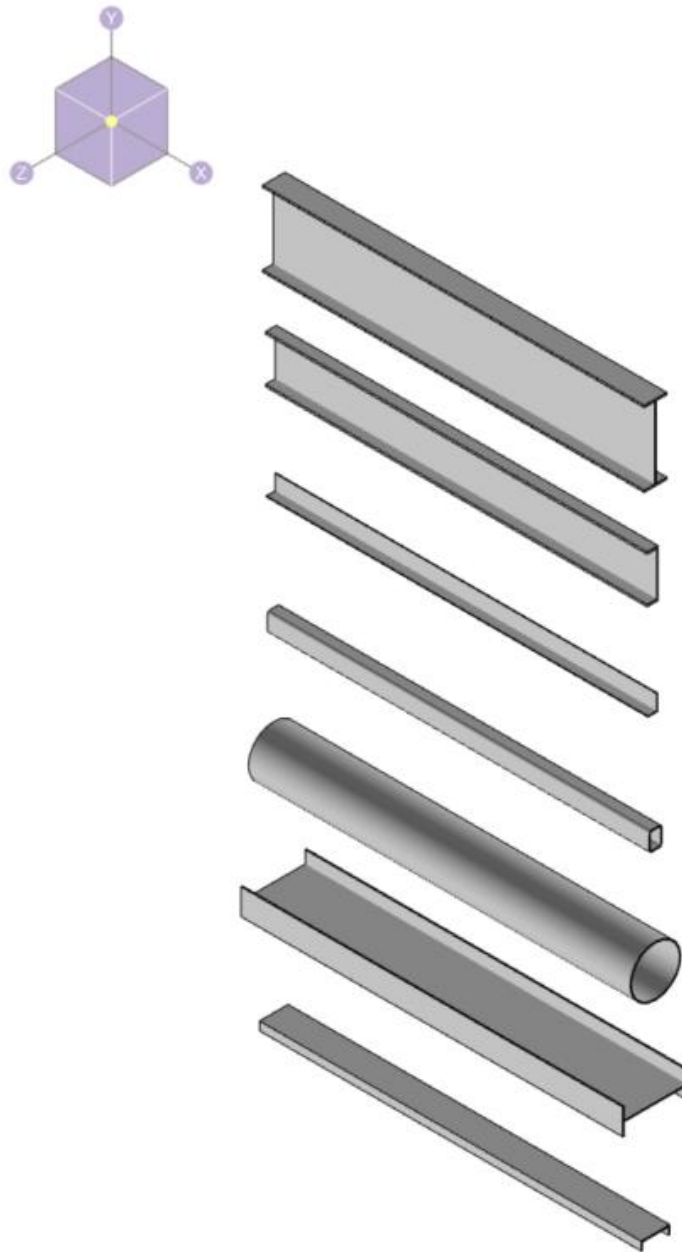


Figure 18.1 – Model View

## Validation Method

In this example we are checking the shear capacity of singly or doubly symmetric members with shear in the plane of the web, single angles, HSS sections, and shear in the weak direction of symmetric shapes.

## Comparison

Example	Shape	Capacity Value (kips)	RISA Value (kips)	AISC Value (kips)	% Difference
G.1	W24x62	$V_{ny}/\Omega$	203.82	204	0.09
G.2	C15x33.9	$V_{ny}/\Omega$	77.605	77.6	0.01
G.3	L5x3x $\frac{1}{4}$	$V_{ny}/\Omega$	16.168	16.2	0.20
G.4	HSS6x4x $\frac{3}{8}$	$V_{ny}/\Omega$	62.105	62.3	0.31
G.5	HSS16x3/8	$V_{ny}/\Omega$	142.132	142	0.09
G.6	W21x48	$V_{nz}/\Omega$	125.756	126	0.19
G.7	C9x20	$V_{nz}/\Omega$	28.312	28.3	0.04

Table 18.1 – Shear Comparison

## Conclusion

As seen in Table 18.1 above, the results are within a reasonable round-off difference from the AISC hand calculation.

# Verification Problem 19

---

## Problem Statement

This model is a collection of members that verifies the AISC 360-22 specification for design members for combined forces from the AISC Design Examples 16<sup>th</sup> edition. Each of these is using the ASD design parameters and is built with the exact specifications from the example problems.

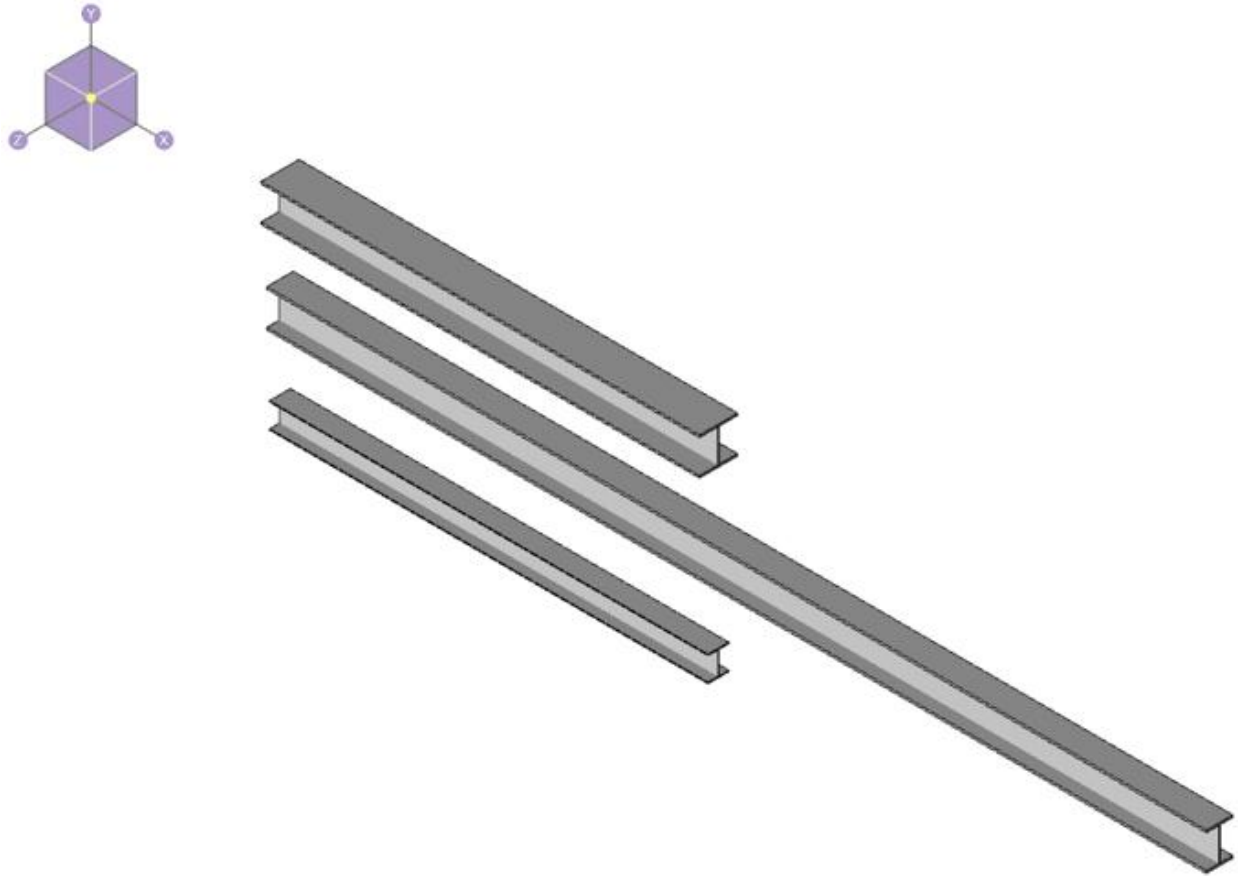


Figure 19.1 – Model View

## Validation Method

In this example we are checking combined forces and torsion of the designed members. Some notes about specific problems:

- Example H.2: RISA does not consider section H2 of the AISC 360-10 specification, so example H.2 was omitted.
- Example H.4: Nodes were added along the length of the member in this example so that P-delta effects would be considered. Example H.4 uses the  $B_1$  amplifier to accomplish this.

## Comparison

Example	RISA UC Max Value	AISC Value	% Difference
H.1	0.930	0.931	0.11
H.3	0.876	0.874	0.23
H.4	0.983	0.982	0.10

Table 19.1 – Comparison

## Conclusion

As seen in Table 19.1 above, the results are within a reasonable difference from the AISC hand calculation.

# Verification Problem 20

---

## Problem Statement

This model will be used to verify the design values for aluminum compressive members (columns).

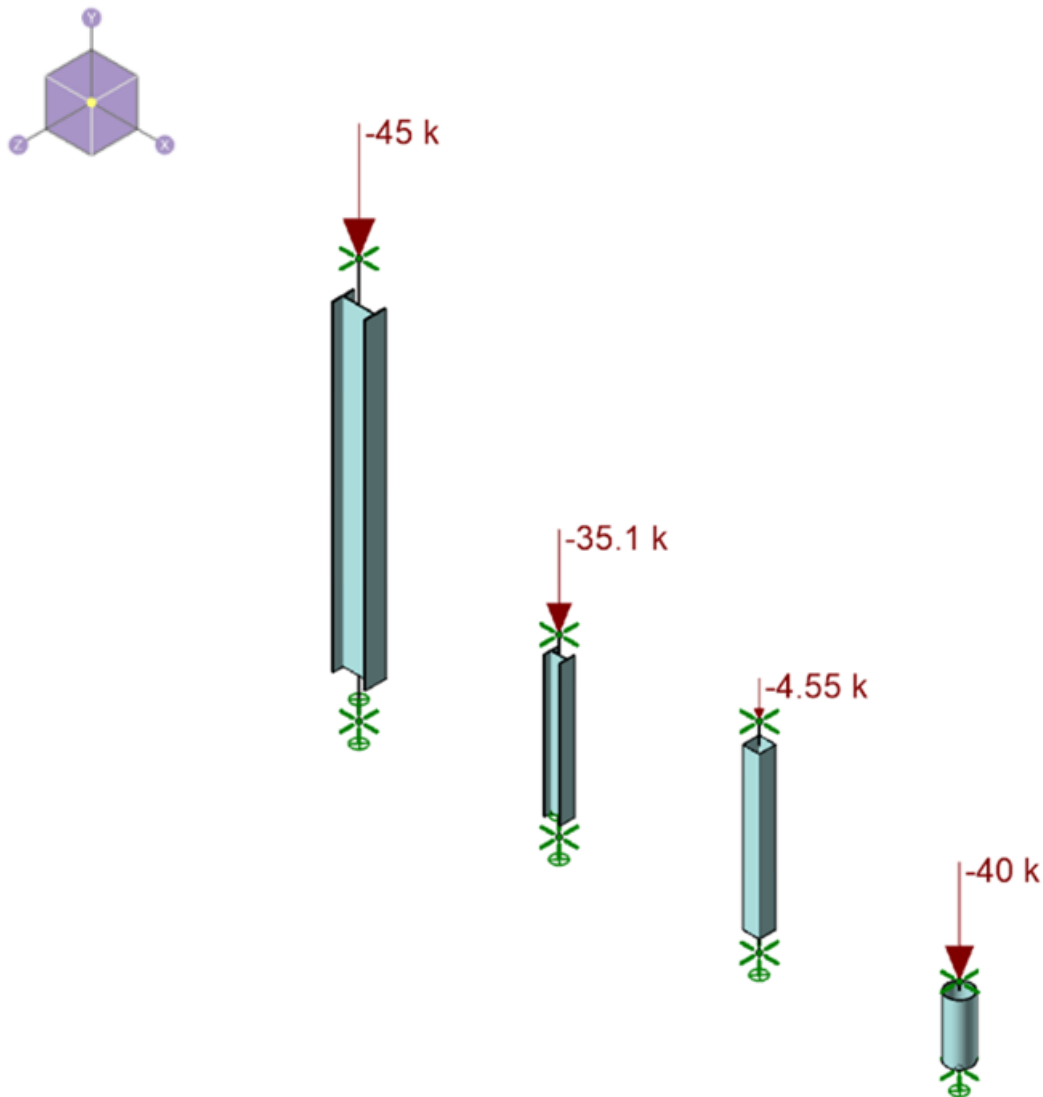


Figure 20.1 – Model View

## Validation Method

The program results will be compared to the design value published in the *2010 Aluminum Design Manual* by the Aluminum Association. These examples were taken from Part VIII of the ADM, examples 9, 11, 12, and 14.

## Comparison

For this model:

	Slenderness	Slenderness Lower Limit	Slenderness Upper Limit	Compressive Strength
	S	S1	S2	Pnc/Ω (k)
RISA Model - Member M1	59.8	-	65.7	66.86
ADM Example 9	28.5	-	66.0	16.70
% Difference	*	-	0.45	*

	Slenderness	Slenderness Lower Limit	Slenderness Upper Limit	Compressive Strength
	S	S1	S2	Pnc/Ω (k)
RISA Model - Member M2	52.9	-	65.7	35.32
ADM Example 11	53.0	-	66.0	35.40
% Difference	0.19	-	0.45	0.23

	Slenderness	Slenderness Lower Limit	Slenderness Upper Limit	Compressive Strength
	S	S1	S2	Pnc/Ω (k)
RISA Model - Member M3	61.5	-	62.2	5.17
ADM Example 12	61.5	-	60.0	5.40
% Difference	0.00	-	3.54**	4.45**

	Slenderness	Slenderness Lower Limit	Slenderness Upper Limit	Compressive Strength
	S	S1	S2	Pnc/Ω (k)
RISA Model - Member M4	8.8	-	65.7	65.76
ADM Example 14	8.7	-	66.0	65.80
% Difference	1.14	-	0.46	0.06

Table 20.1 – Slenderness and Strength Comparisons

As seen in Table 20.1 above, the results are within a reasonable difference from the hand calculations with the few exceptions noted below.

\* Per section E.3 of the Design Manual, RISA is taking the largest  $kL/r$  value per sections E.3.1 & E.3.2. However, it looks like the example is only taking the  $kL/r$  value per section E.3.1. Please see the hand calculations below for further verification of how RISA calculates these values.

\*\* The design example is rounding off by quite a bit in example 14 which is why the % difference is so high. Please see the hand calculations below for an exact verification of how RISA calculates these values.

## Hand Calculations

### **Member M1, Load Combination 1**\_\_\_\_\_

#### Cross Sectional Properties:

$$I_z = 59.7 \cdot \text{in}^4 \quad \text{Moment of Inertia (Strong Axis)}$$

$$I_y = 7.3 \cdot \text{in}^4 \quad \text{Moment of Inertia (Weak Axis)}$$

$$A = 5.26 \cdot \text{in}^2 \quad \text{Area}$$

$$r_z = \sqrt{\frac{I_z}{A}} = 3.369 \text{ in} \quad \text{Radius of Gyration (Strong Axis)}$$

$$r_y = \sqrt{\frac{I_y}{A}} = 1.178 \text{ in} \quad \text{Radius of Gyration (Weak Axis)}$$

$$J = 0.188 \cdot \text{in}^4 \quad \text{Torsional J}$$

$$C_w = 107 \cdot \text{in}^6 \quad \text{Warping Constant}$$

$$\Omega = 1.65$$

#### Unbraced Lengths:

$$K = 1.0$$

$$L_z = 96 \cdot \text{in}$$

#### Material Properties:

$$E = 10100 \cdot \text{ksi}$$

$$G = 3787.5 \cdot \text{ksi}$$

$$F_{cy} = 35 \cdot \text{ksi}$$

#### Slenderness:

Per section E.3,  $KL/r$  shall be taken as the largest slenderness ratio per sections E.3.1 & E.3.2

Per section E.3.1:

$$S_{E31} = \frac{K \cdot L_z}{r_z} = 28.496$$

Per section E.3.2:

$$F_e = \left[ \frac{\left( \pi^2 \cdot E \cdot C_w \right)}{(K \cdot L_z)^2} + G \cdot J \right] \cdot \left[ \frac{1}{(I_z + I_y)} \right] = 27.901 \text{ ksi} \quad (\text{Per eqn E.3-6})$$

$$S_{E32} = \pi \cdot \sqrt{\frac{E}{F_e}} = 59.772 \quad (\text{Per eqn E.3-5})$$



Therefore,

$$S = \max(S_{E31}, S_{E32}) = 59.772$$

Axial Capacity Calculations:

$$B_c = F_{cy} \cdot \left( 1 + \sqrt{\frac{35}{2250}} \right) = 39.365 \text{ ksi} \quad (\text{Per table B.4.2})$$

$$D_c = \left( \frac{B_c}{10} \right) \cdot \sqrt{\frac{B_c}{E}} = 0.246 \text{ ksi} \quad (\text{Per table B.4.2})$$

$$F_c = 0.85 \cdot (B_c - D_c \cdot S) = 20.974 \text{ ksi} \quad (\text{Per eqn E.3-2})$$

$$P_{nc} = \frac{(F_c \cdot A)}{\Omega} = 66.864 \text{ kip} \quad (\text{Per eqn E.3-1})$$

**Member M3, Load Combination 1**-----

Cross Sectional Properties:

$$A = 0.992 \cdot \text{in}^2$$

$$t = 0.063 \cdot \text{in}$$

$$b = 4 \cdot \text{in} - 2 \cdot t = 3.874 \text{ in}$$

Material Properties:

$$F_{cy} = 13 \cdot \text{ksi}$$

$$E = 10100 \cdot \text{ksi}$$

$$\Omega = 1.65$$

Slenderness:

$$k_1 = 0.5 \quad (\text{Per table B.4.3})$$

$$S = \frac{b}{t} = 61.492 \quad (\text{Per section B.5.4.2})$$

$$B_p = F_{cy} \cdot \left[ 1 + \left( \frac{13}{440} \right) \left( \frac{1}{3} \right) \right] = 17.019 \text{ ksi} \quad (\text{Per table B.4.1})$$

$$D_p = \left( \frac{B_p}{20} \right) \cdot \sqrt{\left( \frac{6 \cdot B_p}{E} \right)} = 0.086 \text{ ksi} \quad (\text{Per table B.4.1})$$

$$S_2 = \frac{k_1 \cdot B_p}{1.6 \cdot D_p} = 62.158 \quad (\text{Per section B.5.4.2})$$

Axial Capacity Calculations:

$$F_c = B_p - 1.6 \cdot D_p \cdot S = 8.601 \text{ ksi} \quad (\text{Per section B.5.4.2})$$

$$P_{nc} = \frac{(F_c \cdot A)}{\Omega} = 5.171 \text{ kip}$$

# Verification Problem 21

---

## Problem Statement

This model will be used to verify the design values for aluminum bending members (beams).

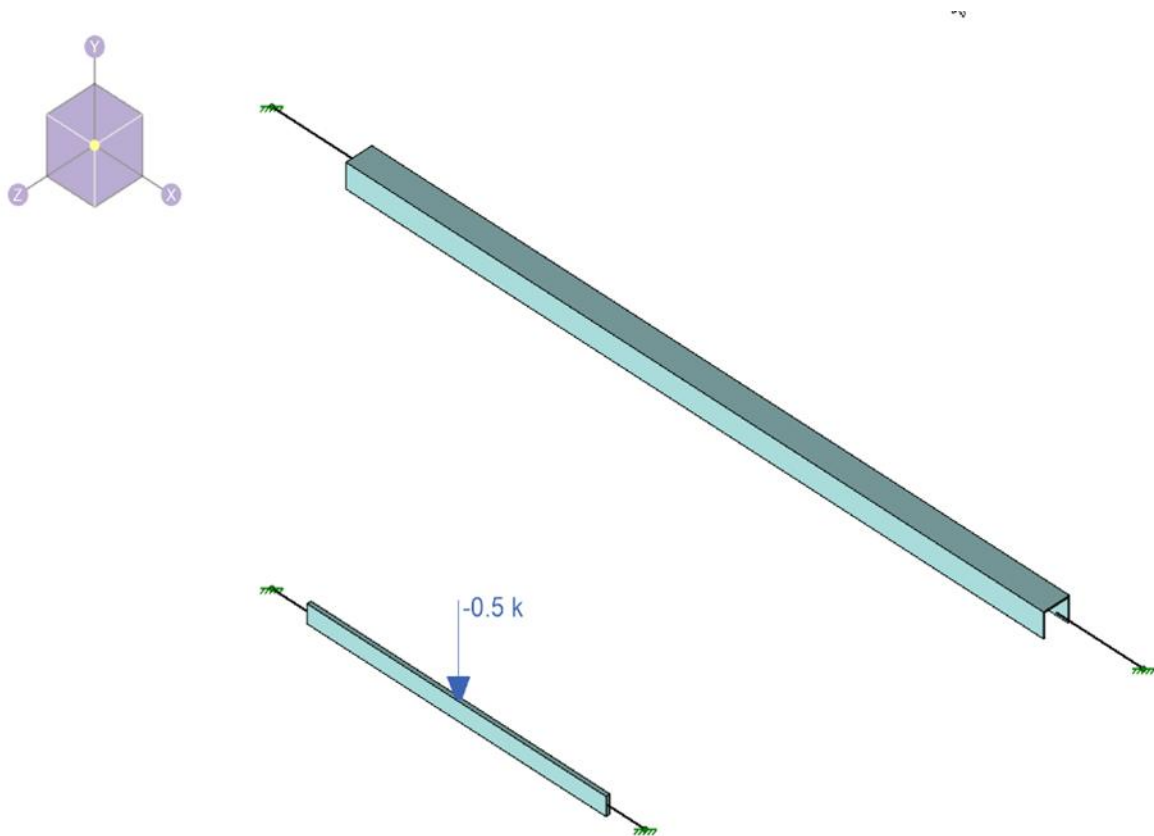


Figure 21.1 – Model View

## Validation Method

The program results will be compared to the design value published in the *2010 Aluminum Design Manual* by the Aluminum Association. These examples were taken from Part VIII of the ADM, examples 19 and 23.

**Note:** For example no. 23, comparisons were only made to the channel shape *without* stiffeners.

## Comparison

For this model:

	Bending Strength about the Strong Axis	Governing Moment Force	Slenderness	Slenderness Upper Limit
	$M_{nz}/\Omega$ (k-in)	M (k-in)	S	S2
RISA Model - Member M1	2.39	2.25	19.6	36
ADM Example 19	2.39*	2.25	19.6	36
% Difference	0.00	0.00	0.00	0.00

	Bending Strength about the Weak Axis	Slenderness	Slenderness Lower Limit	Slenderness Upper Limit
	$M_{ny}/\Omega$ (k-in)	S	S1	S2
RISA Model - Member M2	3.84	15	10.2	23
ADM Example 23	3.81	15	10.2	23
% Difference	0.78	0.00	0.00	0.00

Table 21.1 – Slenderness and Strength Comparisons

As seen in Table 21.1 above, the results are within a reasonable difference from the hand calculations.

\*This value was obtained by multiplying the Tensile Rupture allowable stress value from the example by the section modulus.

# Verification Problem 22

---

## Problem Statement

This problem is a simply-supported reinforced concrete beam model solved using RISA-3D and the result was compared with Example 4-1 in the *Reinforced Concrete Mechanics and Design, 6th Edition* by James K. Wight and James G. MacGregor. The primary use of this problem is to verify the moment capacity for a reinforced concrete beam from RISA-3D versus that obtained by the reference book.

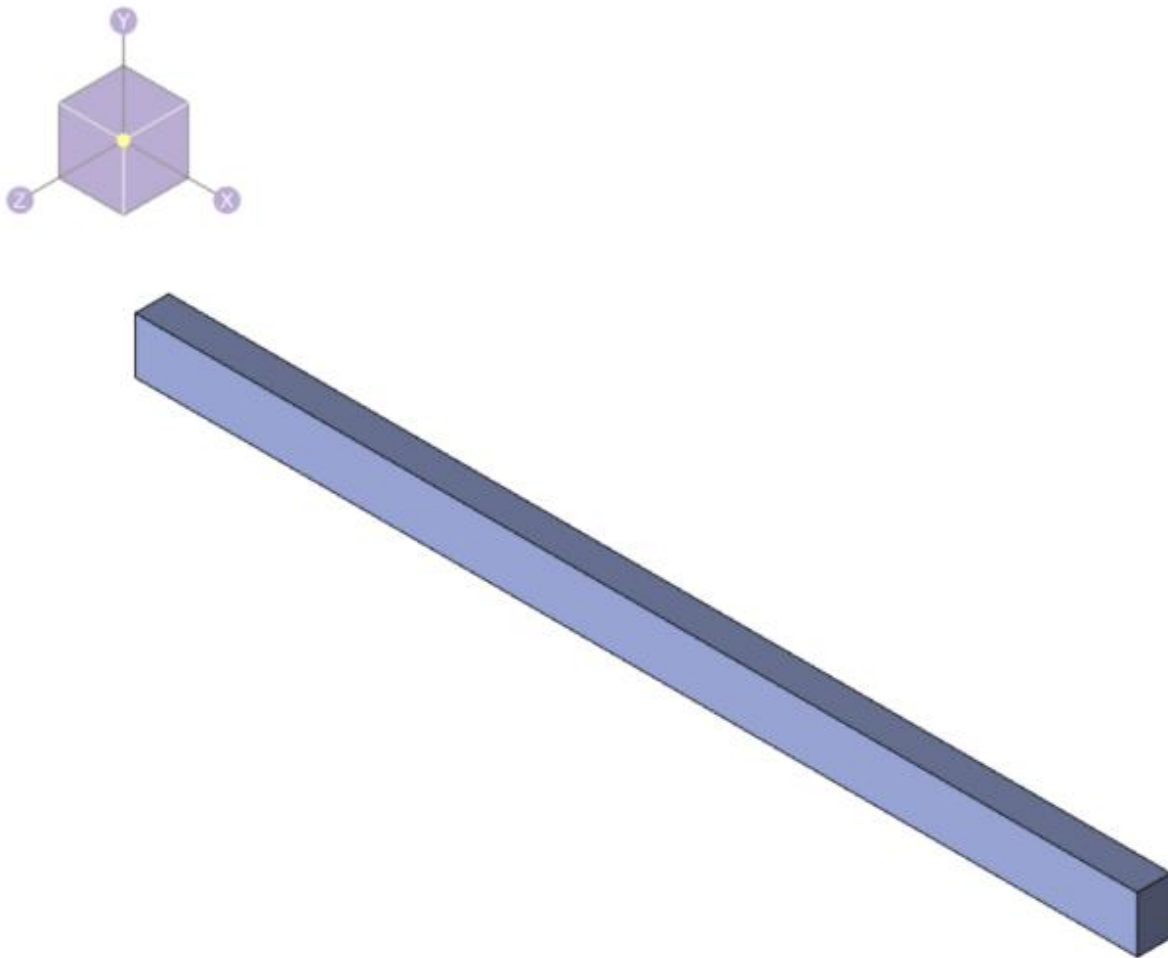


Figure 22.1 – Model View

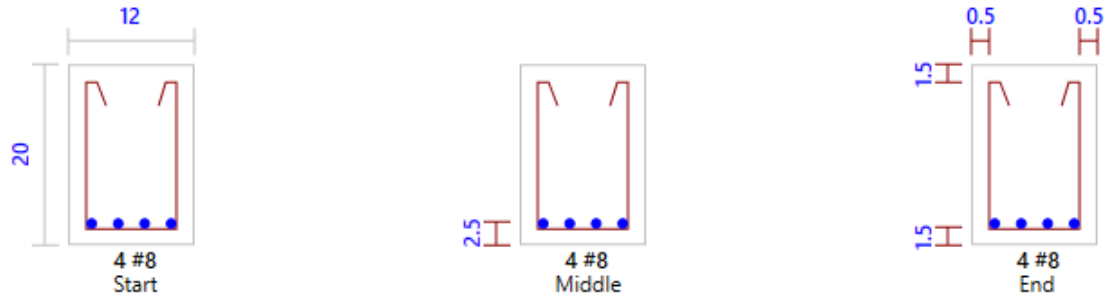


Figure 22.2 – Cross Section of Beam (Units: inch)

Nominal Moment Capacity	RISA-3D	Reference book	% Difference
$M_n(k\text{-ft})$	238.6	240.0	0.6

Table 22.1 – Nominal Moment Capacity Comparison