

# Prestressing Losses and Elongation Calculations

## 1. BACKGROUND

The stresses in a prestressing strand normally vary along its length and decrease with time. The principal factors affecting the distribution of stress along a strand are:

- friction losses during stressing;
- retraction of strand as it seats and locks into the anchorage device (seating loss);
- elastic shortening of concrete, when tendons are not all stressed simultaneously;
- shrinkage of concrete;
- creep of concrete; and,
- relaxation of steel.

Other factors such as changes in temperature and flexing of the structure under loading also affect the stresses in a strand, but these do not necessarily result in a permanent lowering of stress level and are not typically considered as stress losses.

The total prestress loss for unbonded, low-relaxation tendons is typically 20 percent of the jacking stress. A lump sum stress loss of 30 ksi (14%) was assumed for several years for pretensioned members, since there is no friction loss in pretensioning. The development of low-relaxation strands and results of subsequent studies prompted a call for more exact estimates. A rigorous evaluation of stress losses is both time consuming and complex, however. Precise calculations for each tendon are not usually warranted in most residential and commercial buildings; studies have indicated that reliable solutions can be obtained with a number of simplifying assumptions.

The commentary for ACI 318, Chapter 18 states the following:

*“Lump sum values of prestress losses for both pre-tensioned and post-tensioned members which were indicated in pre-1983 editions of the commentary are considered obsolete. Reasonably accurate estimates of prestress losses can be easily calculated in accordance with recommendations in Reference<sup>1</sup> which include considerations of initial stress level ( $0.70f_{pu}$  or higher), type of steel...”*

ACI-318's refers to a study initiated by ACI/ASCE Committee 423, directed by Paul Zia and reported in Concrete International [Zia et al, 1979]. The stress losses due to friction and seating of tendon are based on ACI 318. Research on friction losses and the background to the proposed procedures for their calculation are reported in numerous publications including several listed in the References at the end of this section.

It is assumed that the various factors such as friction, creep, and shrinkage that affect the stress losses are independent from one another. Hence, the loss due to each factor may be computed separately. The total stress loss in a tendon is the sum of the individually calculated losses.

In addition to the stress loss factors discussed above, the effective prestressing in a member may be affected by its connections to other structural members that restrain its movement. These factors are not taken into account in the Friction and Long Term Losses post-processor. They should be accounted for based on rational procedures that consider equilibrium of forces and strain compatibility. Aalami and Barth [1987] discuss the consequences of restraint in buildings.

## 2. STRESS DISTRIBUTION

The stress losses along a tendon are illustrated in **Fig. 2-1**. **Figure 2-1(a)** shows a beam with a continuous tendon stressed at both ends. It is assumed that the left end is stressed first. **Figure 2-1(b)** shows the distribution of stress along the strand during stressing, prior to locking off the strand. The jacking stress is commonly specified at  $0.80f_{pu}$ , where  $f_{pu}$  is the specified ultimate strength of strand. The smooth curve is a simplification of the actual distribution for illustration purposes however. The actual shape of the curve is determined by the tendon profile and friction parameters.

**Figure 2-1(c)** shows the distribution after the strand is locked off at the left end of the beam. Observe that the initial stress is partially lost over a length of strand at the left end marked XL. This is the result of the retraction of the strand at the stressing end while the wedges are being seated. Per ACI 318, Chapter 18, the maximum permissible stress value immediately after lock-off and away from

<sup>1</sup> ACI 423

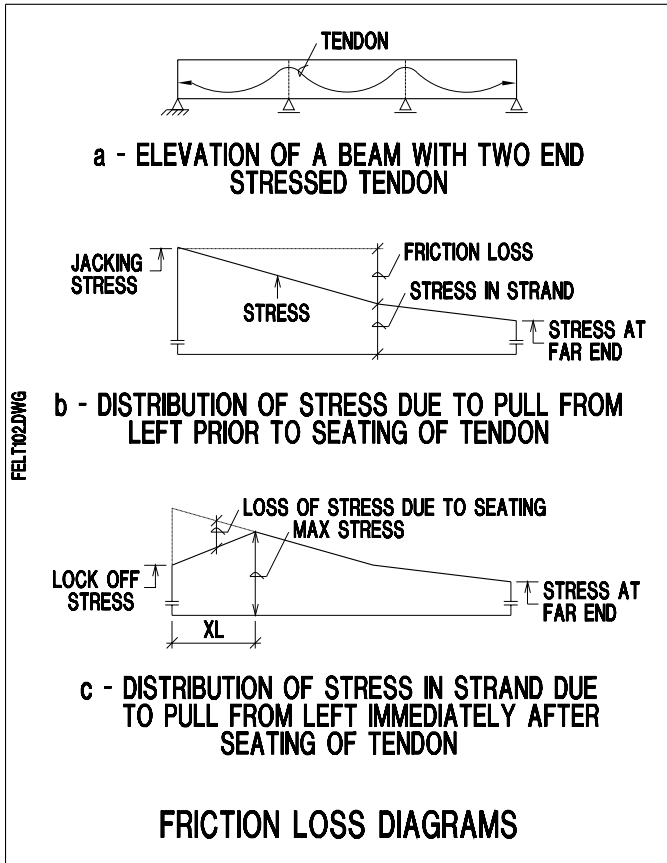


FIGURE 2-1 (cont'd...)

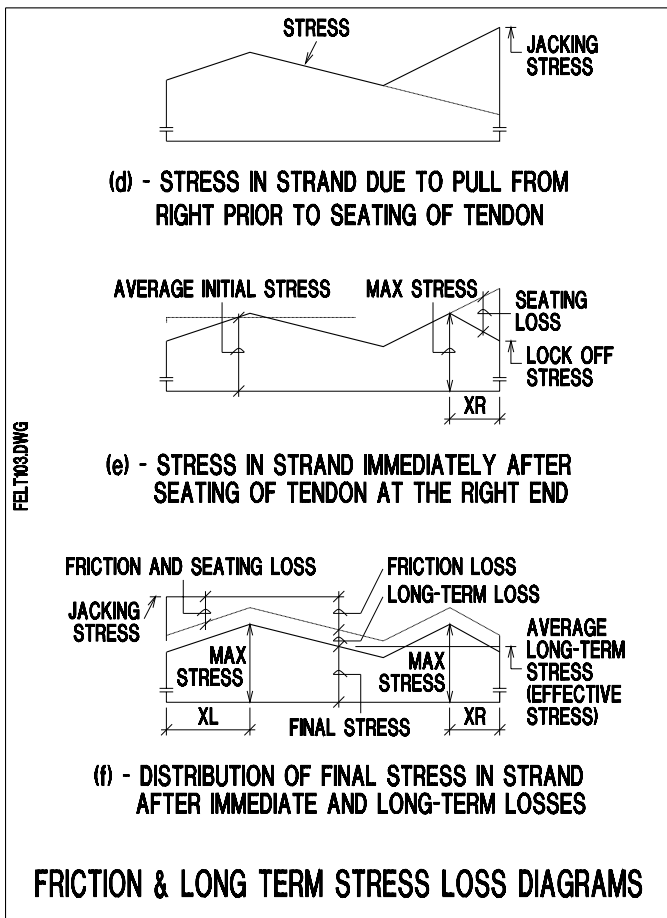


FIGURE 2-1 (continuation)

anchorage device is  $0.74f_{pu}$ . The maximum stress occurs at XL. The maximum permissible stress at the anchorage immediately after seating of the strand is  $0.7f_{pu}$ .

The seating loss, also referred to as anchorage set or draw-in, is typically  $3/8$  to  $1/4$  of an inch (6 to 8 mm). For short strands, and/or larger values of seating loss, the length XL may extend to the far end of the strand. Stressing rams with power seating capability will minimize the seating loss. Note that the retraction of the strand is resisted by the same friction forces that resisted the initial stressing. The stress diagram along length XL thus has the same gradient as the remainder of the curve, but in the opposite direction.

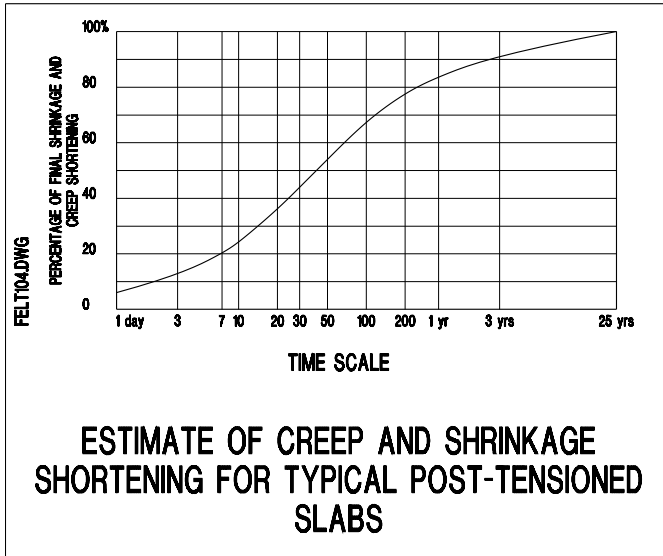
In most cases, jacking of the tendon at right end, **Fig. 2-1(d)**, raises the stresses to about the mid-point of the tendon and the stress diagram will have a second peak at XR **Fig. 2-16(e)**. The distribution of stress immediately after the strand is seated at the right end is shown in **Fig. 2-1(e)**. Note that the lock-off stresses at the left and right ends are not generally the same unless the tendon is symmetrical about its mid-point.

The average initial stress is the average of this stress distribution. This value is used by some designers to calculate the stresses in unbonded post-tensioned structures at the transfer of post-tensioning. Transfer of post-tensioning refers to the loading condition immediately after stressing, prior to the application of live loading and the influences of long-term stress losses. It is also referred to as the "lock-off stress".

As long-term stress losses occur, the stress in strand is reduced along its length. **Figure 2-1(f)** shows a schematic of the stress distribution after all losses have taken place. The following should be noted with respect to the final distribution of stress:

- Long-term stress losses along a tendon are not constant. Even under uniform geometry and exposure conditions, differences in concrete stress along a strand result in non-uniform losses. In the design of commercial buildings, however, it is common practice to calculate a representative long-term loss value for the entire member when **unbonded** tendons are used. The average precompression in concrete is used to calculate the representative stress loss. The average precompression is calculated using the effective prestressing force and gross cross-section of concrete. In **bonded** (grouted) tendon construction, long-term losses are strictly a function of concrete strain at location of tendon along the length of member.
- Long-Term stress losses are obviously a function of time. The relationships developed by the ACI/ASCE committee refer to a time at which over 90 percent of the losses have taken place. For common commercial

buildings this period is between 2 and 2 1/2 years. The stress loss rates for shrinkage, creep and relaxation are not the same however. The curve shown in **Fig. 2-2** may be used as a first approximation to estimate the combined stress losses for concrete at earlier ages. This diagram is compiled from the combined effects of shrinkage and creep using data from the PCI Design Handbook [1999].



**FIGURE 2-2**

The stress diagram computed from the friction formulas given in ACI 318 and shown in the **Fig. 2-2** represent the maximum possible stress gradient attainable from the friction coefficients. The diagram is constructed with the maximum gradient at all points. With unbonded strands, flexing of the member due to applied loading, temperature changes, shrinkage and creep can only reduce the stress gradient. Thus there could actually be a flattening of the diagram toward a more uniform stress distribution along the length of the tendon. This is the premise for the use of “effective” stress in design of post-tensioned members reinforced with unbonded tendons. There do not appear to be any conclusive studies that would quantify the extent of the stress redistribution however.

If a final **effective force design** approach is used, the outcome of the design is an effective force to be provided by post-tensioning. The **effective force** is the value that is shown on the structural drawings and in the calculations. The question of whether the effective force is based on average stresses, local stresses, or other considerations is not applicable during design.

At the shop drawing preparation phase, the effective forces must be replaced by the number of strands. In theory, the actual stresses in the strand at each location should be used to arrive at the number of strands required at that location. Because of the lack of information, and the complexity of this approach, however, an effective stress is typically used when designing commercial buildings with unbonded tendons. The effective stress is

the average initial stress (**Fig. 2-1(e)**), minus a representative long-term stress loss value calculated for the entire member. Some engineers refer to the effective stress as the design stress.

When the design is done using a **system bound** approach, the structural calculations are preceded by a friction and long-term loss computation using parameters particular to the post-tensioning supplier, such as friction values. The structural calculations can thus determine the number and location of the strands. In this case, the calculation of the design stress is of prime importance to the structural designer. The ACI code specifies that the stresses used in structural computations should be derived with due considerations to immediate and long-term losses. Unless satisfactory research shows otherwise, the use of an effective stress does not seem justified in a system bound design approach.

### 3. FRICTION AND SEATING LOSS CALCULATIONS

#### 3.1 Stress Loss Due to Friction

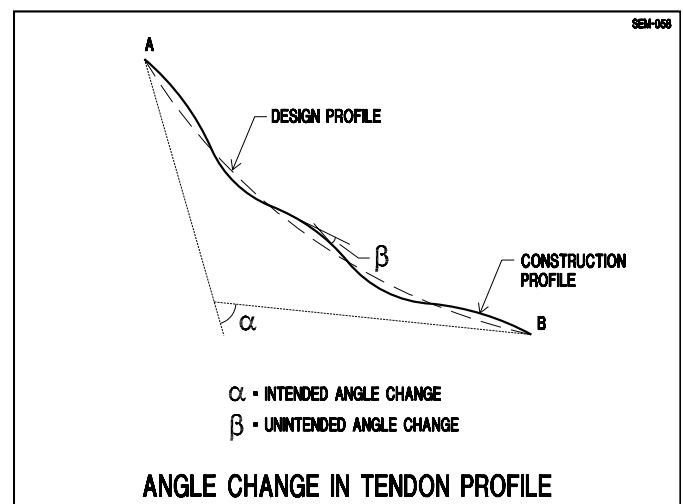
The stress at any point along a strand is related to the jacking stress through the following relationship:

$$P_x = P_j * e^{-(\mu\alpha + Kx)}$$

Where,

- $P_x$  = stress at distance x from the jacking point;
- $P_j$  = stress at jacking point;
- $\mu$  = coefficient of angular friction;
- $\alpha$  = total angle change of the strand in radians from the stressing point to distance x;
- x = distance from the stressing point; and,
- K = wobble coefficient of friction expressed in radians per unit length of strand (rad/unit length<sup>2</sup>).

**Figure 3.1-1** illustrates the design intended change in angle “ $\alpha$ ” and the unintended change in angle “ $\beta$ ” dependent on construction practice.



**FIGURE 3.1-1**

<sup>2</sup> The dimension of wobble coefficient K includes the coefficient of friction. K is  $\mu$ \*(average of unintended change in angle per unit length of tendon)

**Table 3.1-1**, extracted from the PTI Manual [1990], gives friction coefficients for common strand and duct materials. Note that unbonded, monostrand tendons are referred to as ‘Greased and Wrapped’ in this table. A similar table is given in ACI-318. The post-tensioning supplier should be consulted for friction coefficients of duct and coating materials not shown.

**TABLE 3.1-1 FRICTION COEFFICIENTS FOR POST-TENSIONING TENDONS**

RANGE OF VALUES			
Type of Tendon 1	Wobble coefficient 2		Curvature coefficient $\mu$ 3
	K (rad. per ft)	K (rad. per m)	
Flexible tubing; (bonded)			
non-galvanized	0.0005-0.0010	0.0016-0.0033	0.18-0.26
galvanized	0.0003-0.0007	0.0010-0.0023	0.14-0.22
Rigid thin wall tubing; (bonded)			
non-galvanized	0.0001-0.0005	0.0003-0.0016	0.20-0.30
galvanized	0.0000-0.0004	0.0000-0.0013	0.16-0.24
Greased and wrapped (unbonded)	0.0005-0.0015	0.0016-0.0049	0.05-0.15
b - RECOMMENDED			
Type of Tendon 1	Wobble coefficient 2		Curvature coefficient $\mu$ 3
	K (rad. per ft)	K (rad. per m)	
Flexible tubing; (bonded)			
non-galvanized	0.0075	0.0246	0.22
galvanized	0.0005	0.0016	0.18
Rigid thin wall tubing; (bonded)			
non-galvanized	0.0003	0.0010	0.25
galvanized	0.0002	0.0007	0.20
Greased and wrapped (unbonded)	0.0010	0.0033	0.07

### 3.2 Elongation

**Figure 2-1(a)** shows a typical post-tensioning tendon profile. When the jacking force,  $P_j$ , is applied at the stressing end, the tendon will elongate in accordance with the following formula:

$$\Delta = \int P_x dx / (A * E_s)$$

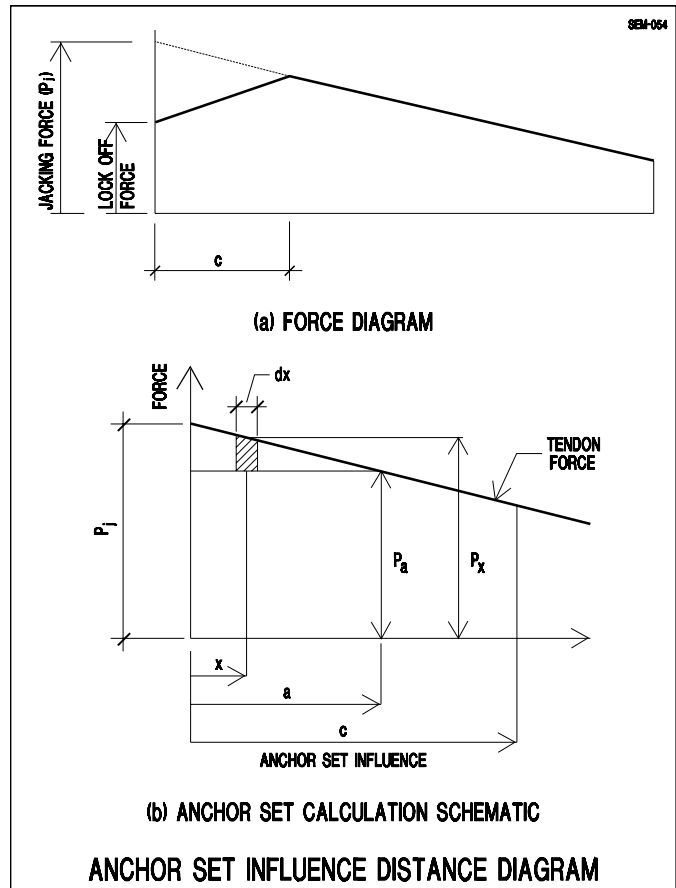
Where,

- A = cross-sectional area of the tendon;
- dx = the element of length along tendon;
- $E_s$  = modulus of elasticity of the prestressing steel (typically taken as either 28000 or 28500 ksi) (193054 MPa or 196502 MPa);
- $P_x$  = tendon force at distance x from the jacking end; and,
- $\Delta$  = calculated elongation.

This elongation will be resisted by friction between the strand and its sheathing or duct, however. As a result of this friction, there will be a drop in the force in the tendon with distance from the jacking end. The friction is comprised of two effects: curvature friction which is a function of the tendon’s profile, and wobble friction which is the result of minor horizontal or vertical angular deviations from the design-intended profile. Curvature friction is greatest when there are short spans with fairly large changes in profile.

### 3.3 Seating Losses

After they are stressed, tendons are typically anchored with two-piece conical wedges. The strand retracts when it is released and pulls the wedges into the anchorage device; this forces the wedges together and locks the strand in place. The stress loss due to seating is somewhat hard to calculate because the loss in elongation is fairly small (it depends on both the jack and jacking procedure.) In addition, the loss in elongation (referred to as anchor set, or draw-in) is resisted by friction much as the elongation itself is resisted by friction.



**FIGURE 3.3-1**

Calculation of the stress loss is typically done as an iterative process. Refer to **Fig. 3.3-1a**. After anchor set, the region of stress distribution affected by the loss in stress is the mirror image of the associated stress curve under jacking force. The extent of the influence of anchor set is shown as “c.” For the calculation of distance “c” an anchor set influence length “a” is chosen (**Fig. 3.3-1b**). An elongation  $\Delta_a$  for the selected distance “a” is calculated using the formula:

$$\Delta_a = (1/A * E_s) * \int (P_x - P_a) * dx$$

Where,

- A = tendon cross sectional area,
- dx = element of distance along tendon length
- $E_s$  = modulus of elasticity of tendon,
- $P_a$  = force in tendon under jacking stress at the assumed anchor set distance “a”,

- $P_x$  = force in tendon at distance “x” from the stressing end,
- $\Delta_a$  = elongation associated with the assumed anchor set influence length “a”.

The anchor set length is adjusted until the calculated  $\Delta_a$  is reasonably close to the seating loss. The integral is carried out for each stressing end.

The average stress is calculated as the area under the stress diagram divided by the length of the tendon. Note that the slope of the “post-seating” stress line is the inverse of the initial stress loss line. The elongation for the first stressing is the average stress in the tendon after the first stressing, divided by the modulus of elasticity of the strand. The elongation for the second stressing is the average stress in the tendon divided by the modulus of elasticity, minus the first elongation.

#### 4. LONG-TERM STRESS LOSS ESTIMATE

For common structures and conditions, simplified equations are used to estimate the stress losses due to prestressing. The equations are based on the work of ACI-ASCE Committee 423 [1979], PCI [1999]. The equations enable the designer to estimate the various types of prestress loss, rather than using a lump sum value. It is believed that these equations, intended for practical design applications, provide fairly realistic values of normal design conditions. For unusual design situations and special structures, more detailed analysis may be warranted Aalami [1998].

$$TL = ES + CR + SH + RE$$

Where,

- CR = stress loss due to creep,
- ES = stress loss due to elastic shortening,
- RE = stress loss due to relaxation in prestressing steel,
- SH = stress loss due to shrinkage of concrete, and
- TL = total loss of stress.

##### 4.1 Elastic Shortening of Concrete (ES)

Elastic shortening refers to the shortening of the concrete member as the post-tensioning force is applied. If there is only one tendon in a member, there will be no loss due to elastic shortening since the elastic shortening will have occurred before the tendon is locked into place. Generally, however there will be several tendons in a member. As each tendon is tensioned, there will be a loss of prestress in the previously tensioned tendons due to the elastic shortening of the member.

Since an **unbonded** tendon can slide within its sheathing, it typically does not experience the same stress-induced strain changes as the concrete

surrounding it. For this reason, the average compressive stress in the concrete,  $f_{cpa}$ , is typically used to calculate prestress losses due to elastic shortening and creep for unbonded tendons. This relates these prestress losses to the average member strain rather than the strain at the point of maximum moment.

The equation given for calculating elastic shortening for unbonded tendons is:

$$ES = K_{es} (E_s/E_{ci}) f_{cpa}$$

Where,

- ES = is the elastic modulus of the prestressing steel;
- $E_{ci}$  = is the elastic modulus of the concrete at time of prestress transfer;
- $K_{es}$  = 1.0 for pre-tensioned members
- $K_{es}$  = 0.5 for post-tensioned members when tendons are tensioned in sequential order to the same tension. With other post-tensioning procedures,  $K_{es}$  may vary from 0 to 0.5; and,
- $f_{cpa}$  = average compressive stress in the concrete along the length of the member at the center of gravity (CGS) of the tendon immediately after the prestress transfer. Note that the stress at the CGS is larger than the average compression in a member.

At the time they are stressed, the ducts in which **post-tensioned bonded** tendons are housed have usually not been grouted. Thus, the elastic shortening equations for unbonded tendons would apply to these tendons as well.

For **pre-tensioned bonded** members the following relationship applied.

$$ES = K_{es} (E_s/E_{ci}) * f_{cir}$$

where,

- $K_{es}$  = 1.0 for pretensioned members;
- $f_{cir}$  = net compressive stress in concrete at center of gravity of tendons immediately after prestress has been applied to concrete;

- $f_{cir} = K_{cir} * f_{cpi} - f_g$
- $K_{cir} = 0.9$  for pre-tensioned members;
- $K_{cir} = 1.0$  for post-tensioned members;
- $f_{cpi}$  = Stress in concrete at center of gravity of tendons due to prestressing forces immediately after prestress has been applied;

$$f_{cpi} = ( P_i/A_g + P_i * e^2/I_g )$$

where,

- $P_i$  = initial prestressing force (after anchorage seating loss);

$A_g$  = area of gross concrete section at the cross section considered  
 $e$  = eccentricity of center of gravity of tendons with respect to center of gravity of concrete at the cross section considered;  
 $I_g$  = moment of inertia of gross concrete section at the cross section considered;  
 $f_g$  = stress in concrete at center of gravity of tendons due to weight of structure at time prestress is applied (positive if tension).

$$f_g = M_g * e / I_g$$

where,

$M_g$  = bending moment due to dead weight of prestressed member and any other permanent loads in place at time the prestressed member is lifted off its bed. There can be an increase in tendon force at the section considered, if pre-tensioning is not adequate to give the member an upward camber (tension due to self weight ( $f_g$ ) greater than compression due to prestressing ( $f_{cpi}$ )).

Hence

$$f_{cir} = K_{cr} (P_i / A_g + P_i * e^2 / I_g) - M_g * e / I_g$$

## 4.2 Creep of Concrete (CR)

Over time, the compressive stress induced by post-tensioning causes a shortening of the concrete member. This phenomenon, the increase in strain due to a sustained stress, is referred to as creep. Loss of prestress due to creep is proportional to the net permanent compressive stress in the concrete. The initial compressive stress induced in the concrete at transfer is subsequently reduced by the tensile stress resulting from self-weight and superimposed dead load moments.

For members with **unbonded** tendons, the equation is:

$$CR = K_{cr} (E_s / E_c) f_{cpi}$$

For members with **bonded** and **pretensioned** tendons, the equation is:

$$CR = K_{cr} (E_s / E_c) (f_{cir} - f_{cds})$$

Where:

$E_c$  = elastic modulus of the concrete at 28 days;

$f_{cds}$  = stress in the concrete at the CGS of the tendons due to all sustained loads that are applied to the member after it has been stressed; and

$K_{cr}$  = maximum creep coefficient; 2.0 for normal weight concrete; 1.6 for sand-lightweight concrete.

The difference in the equations is due to the fact that unbonded tendons do not experience the same strains as the surrounding concrete. The prestress loss due to creep is thus more logically related to the average stress in the concrete. With bonded tendons however, once the duct is grouted the shortening of the concrete member due to creep will result in a comparable shortening (loss of elongation) in the tendon. The same applies to pretensioned members.

## 4.3 Shrinkage of Concrete (SH)

In the calculation of prestress losses, shrinkage is considered to be entirely a function of water loss. Shrinkage strain is thus influenced by the member's volume/surface ratio and the ambient relative humidity. The effective shrinkage strain,  $\epsilon_{sh}$  is obtained by multiplying the basic ultimate shrinkage strain,  $(\epsilon_{sh})_{ultimate}$ , taken as  $550 \times 10^{-6}$  for normal conditions, by the factors  $(1 - 0.06 V/S)$  to allow for member dimension, and  $(1.5 - 0.015RH)$  for ambient humidity.

$$\begin{aligned} \epsilon_{sh} &= 550 \times 10^{-6} (1 - 0.06 * V/S) (1.5 - 0.015 * RH) \\ &= 8.2 \times 10^{-6} (1 - 0.06 * V/S) (100 - RH) \end{aligned}$$

The equation for losses due to shrinkage is:

$$\begin{aligned} SH &= 8.2 \times 10^{-6} K_{sh} E_s (1 - 0.06 V/S) (100 - RH) \\ &\quad \text{(US Units)} \\ &= 8.2 \times 10^{-6} K_{sh} E_s (1 - 0.00236 V/S) (100 - RH) \\ &\quad \text{(SI Units)} \end{aligned}$$

Where:

$V/S$  = volume to surface ratio;

$RH$  = relative humidity (percent), see **Fig. 4.3-1** for USA;

$K_{sh}$  = a factor that accounts for the amount of shrinkage that will have taken place before the prestressing is applied.

For post-tensioned members,  $K_{sh}$  is taken from **Table 4.3-1**.

For pretensioned members,  $K_{sh} = 1$ ;

**TABLE 4.3-1 SHRINKAGE CONSTANT  $K_{sh}$**

DAYS*	1	3	5	7	10	20	30	60
$K_{sh}$	0.92	0.85	0.80	0.77	0.73	0.64	0.58	0.45

\* DAYS refers to the time from the end of moist curing to the application of prestressing.

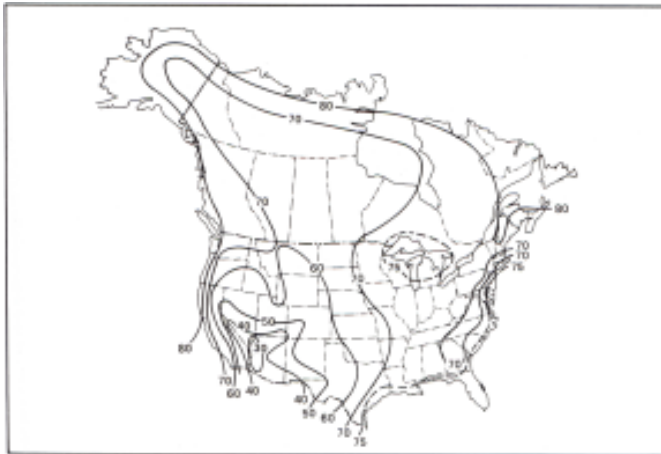
For stressing more than 60 days after curing, a value of 0.45 is assumed.

In structures that are not moist cured,  $K_{sh}$  is typically based on the time when the concrete was cast. It should be noted that in most structures, the prestressing is applied within five days of casting the concrete, whether or not it is moist-cured.

If the ultimate shrinkage value of the concrete being used is different from 550 microstrain used in the above relationships, the calculated stress loss must be adjusted by the following coefficient.

$$SH_{adjusted} = SH [(\epsilon_{sh})_{ultimate}/550]$$

Note that the effective shrinkage strain is zero under conditions of 100% relative humidity, i.e. if the concrete is continuously submerged in water.



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**FIGURE 4.3-1**

#### 4.4 Relaxation of Tendon (RE)

Relaxation is defined as a gradual decrease of stress in a material under constant strain. In the case of steel, it is the result of a permanent alteration of the grain structure. The rate of relaxation at any point in time depends on the stress level in the tendon at that time. Because of other prestress losses, there is a continual reduction of the tendon stress which causes a corresponding reduction in the relaxation rate.

The equation given for prestress loss due to relaxation of the tendons is:

$$RE = [K_{re} - J*(SH + CR + ES)]*C$$

Where,  $K_{re}$  and  $J$  are a function of the type of steel and  $C$  is a function of both the type of steel and the initial stress level in the tendon ( $f_{pi}/f_{pu}$ ).

**Table 4.4-1** gives values of  $K_{re}$  and  $J$  for different types of steel. The factor  $J$  accounts for the reduction in tendon stress due to other losses. As can be seen, the relaxation of low-relaxation tendons is approximately one-quarter that of stress-relieved tendons.

**Table 4.4-2** gives values for  $C$ . The values for stress-relieved and low-relaxation tendons are different because the yield stress for low relaxation tendons is higher than that of the same grade stress-relieved tendons. Although ACI allows a stress of  $0.74 f_{pu}$  along the length of the tendon immediately after prestress transfer, the stress at post-tensioning

**TABLE 4.4-1 STRESS RELAXATION CONSTANTS  $K_{re}$  AND  $J$**

	Grade and type*	$K_{re}$			$J$
		US (psi)	SI (MPa)	MKS (Kg/cm <sup>2</sup> )	
STRESS RELIEVED	270 strand or wire	20000	137.90	1406.14	0.15
	250 strand or wire	18500	127.55	1300.68	0.14
	240 wire	17600	121.35	1237.40	0.13
	235 wire	17600	121.35	1237.40	0.13
	160 bar	6000	41.37	421.84	0.05
	145 bar	6000	41.37	421.84	0.05
LOW RELAXATION	270 strand	5000	34.47	351.54	0.040
	250 wire	4630	31.92	325.52	0.037
	240 wire	4400	30.34	309.35	0.035
	235 wire	4400	30.34	309.35	0.035

\*In accordance with ASTM A416-74, ASTM A421-76, ASTM A722-75.

**TABLE 4.4-2 STRESS RELAXATION CONSTANT  $C$**

$f_{pi}/f_{pu}$	Stress Relieved Strand or Wire	Stress Relieved Bar or Low Relaxation Strand or Wire
0.80		1.28
0.79		1.22
0.78		1.16
0.77		1.11
0.76		1.05
0.75	1.45	1.00
0.74	1.36	0.95
0.73	1.27	0.90
0.72	1.18	0.85
0.71	1.09	0.80
0.70	1.00	0.75
0.69	0.94	0.70
0.68	0.89	0.66
0.67	0.83	0.61
0.66	0.78	0.57
0.65	0.73	0.53
0.64	0.68	0.49
0.63	0.63	0.45
0.62	0.58	0.41
0.61	0.53	0.37
0.60	0.49	0.33

anchorage and couplers is limited to  $0.70 f_{pu}$ . In the absence of more exact calculations, the ratio ( $f_{pi}/f_{pu}$ ) is typically taken as 0.70 for unbonded post-tensioning. With very short tendons however, the loss due to

anchor set may be such that ( $f_{pi}/f_{pu}$ ) is considerably lower.

For values of ( $f_{pi}/f_{pu}$ ) outside of what is given in this table, the following is assumed:

Stress-relieved strand and wire:

For  $0.00 < (f_{pi}/f_{pu}) < 0.60$ ,  $C =$  linear between 0 and 0.49  
 For  $0.75 < (f_{pi}/f_{pu}) < 0.95$ ,  $C = 1.75$

Stress-relieved bar and low-relaxation strand and wire:

For  $0.00 < (f_{pi}/f_{pu}) < 0.60$ ,  $C =$  linear between 0 and 0.33  
 For  $0.80 < (f_{pi}/f_{pu}) < 0.95$ ,  $C = 1.36$

## 5. EXAMPLES

### 5.1 Friction and Long-Term Stress Losses of An Unbonded Post-Tensioned Slab

#### Structure

- Geometry  
 Six span one way slab spanning 18'-0" (5.49 m) between 14 in. x 34 in. (356x864mm) cast in place concrete beams (Fig 5.1-1).  
 Thickness of slab = 5 in (127mm)

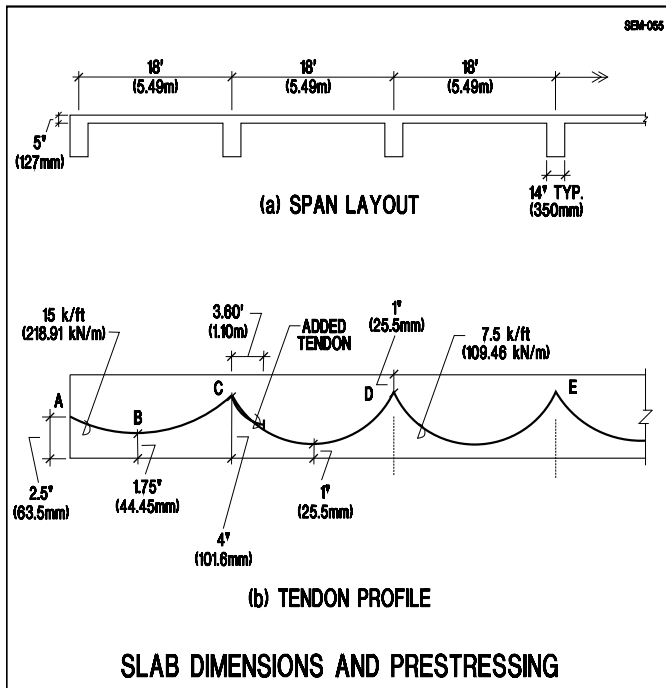


FIGURE 5.1-1

- Material Properties
  - o Concrete:
    - Compressive strength,  $f'_c$   
 = 4000 psi (27.58 MPa)
    - Weight  
 = 150 pcf (2403 kg/m<sup>3</sup>)

Modulus of Elasticity  
 = 3604 ksi (24849 MPa)  
 Age of Concrete at stressing  
 = 3 days  
 Compressive strength at stressing,  $f'_{ci}$   
 = 1832 psi (12.63 MPa)

o Prestressing:  
 Low Relaxation, Unbonded System  
 Strand Diameter = 1/2 in (13 mm)  
 Strand Area = 0.153 in<sup>2</sup> (98mm<sup>2</sup>)  
 Modulus of Elasticity  
 = 28000 ksi (193054 MPa)  
 Coefficient of angular friction,  
 $\mu = 0.07$   
 Coefficient of wobble friction,  
 $K = 0.0014$  rad/ft (0.0046 rad/m)  
 Ultimate strength of strand,  
 $f_{pu} = 270$  ksi (1862MPa)  
 Ratio of jacking stress to strand's  
 ultimate strength = 0.8  
 Anchor set = 0.25 in (6.35 mm)

- Loading:
  - Dead load = self weight + 5 psf  
 (allowance for curbs, lighting,  
 drainage etc)  
 = (5/12) \* 150 + 5 = 68 psf  
 (3.26 kN/m<sup>2</sup>)
  - Live load = 50 psf (2.39 kN/m<sup>2</sup>)

#### A. Friction and Seating Loss

##### i. Friction Loss

Considering only the left half span since the tendon is symmetrical about the mid length and stressed simultaneously from both ends.

Stress at distance  $x$  from the jacking point,  
 $P_x = P_j * e^{-(\mu\alpha + Kx)}$

where,

$P_j$  = Stress at jacking point,  
 =  $0.8 * f_{pu} = 0.8 * 270 = 216$  ksi  
 (1489.28 MPa)

$x$  = distance from the stressing point  
 $\alpha$  = Change of angle in strand  
 (radians) from the stressing point  
 to distance  $x$

Calculation of  $\alpha$ :

Span 1:

$$C = L * \sqrt{(a/b)} / (1 + \sqrt{(a/b)})$$

$$= 18 * \sqrt{(0.75/2.25)} / (1 + \sqrt{(0.75/2.25)})$$

$$= 6.59 \text{ ft (2.01 m)}$$

AB:

$$\alpha = 2 * e / L = 2 * (0.75/12) / 6.59$$

$$= 0.019 \text{ rad}$$



BC:  
 $\alpha = 2 * e / L = 2 * (2.25/12) / 11.41$   
 $= 0.033 \text{ rad}$

$(216 - 203.90) / 36 = x / 20$   
 $x = 6.72 \text{ ksi (46.33 MPa)}$

Span 2:  
 CD:  
 $\alpha = 4 * e / L = 4 * (3/12) / 18$   
 $= 0.056 \text{ rad}$

Seating loss,  $a = (2 * 6.72 * 20 * 12) / (2 * 28000)$   
 $= 0.06 \text{ in (1.52 mm)}$

This is very much less than the given seating loss, 0.25 in

Span 3:  
 DE:  
 $\alpha = 4 * e / L = 4 * (3/12) / 18$   
 $= 0.056 \text{ rad}$

The following table summarizes the stress calculations at supports and first span midpoint, followed by detailed calculation for midpoint.

Location	$\delta x$ , ft (m)	X, ft (m)	$\delta\alpha$ (rad)	$\alpha$ (rad)	$\mu\alpha + Kx$	$e^{-(\mu\alpha + Kx)}$	$P_x$ , ksi (MPa)	Loss, ksi (MPa)
A	0	0	0	0	0	1	216 (1489.28)	0
B	6.59 (2.01)	6.59 (2.01)	0.019	0.019	0.011	0.989	213.62 (1472.87)	2.38 (16.41)
C	11.41 (3.48)	18.00 (5.49)	0.033	0.052	0.029	0.971	209.74 (1446.12)	3.45 (23.79)
D	18.00 (5.49)	36.00 (10.97)	0.056	0.108	0.058	0.944	203.90 (1405.85)	5.84 (40.27)
E	18.00 (5.49)	54.00 (16.46)	0.056	0.164	0.087	0.917	198.07 (1365.65)	5.83 (40.20)

**Detailed Calculation**

$x = 6.59 \text{ ft (2.01m)}$   
 $\alpha = 0.019 \text{ rad,}$   
 $\mu = 0.07$   
 $K = 0.0014 \text{ rad/ft (0.0046 rad/m)}$   
 $P_s = 0.8 * f_{pu} = 0.8 * 270$   
 $= 216 \text{ ksi (1489.28MPa)}$   
 $P_{mid} = P_s * e^{-(\mu\alpha + Kx)}$   
 $= 216 * e^{-(0.07 * 0.019 + 0.0014 * 6.59)}$   
 $= 213.62 \text{ ksi (1472.87 MPa)}$   
 Loss = 216 - 213.62 = 2.38 ksi (16.41MPa)

**ii. Seating Loss**

Stress loss due to seating can be calculated from the following relationship,

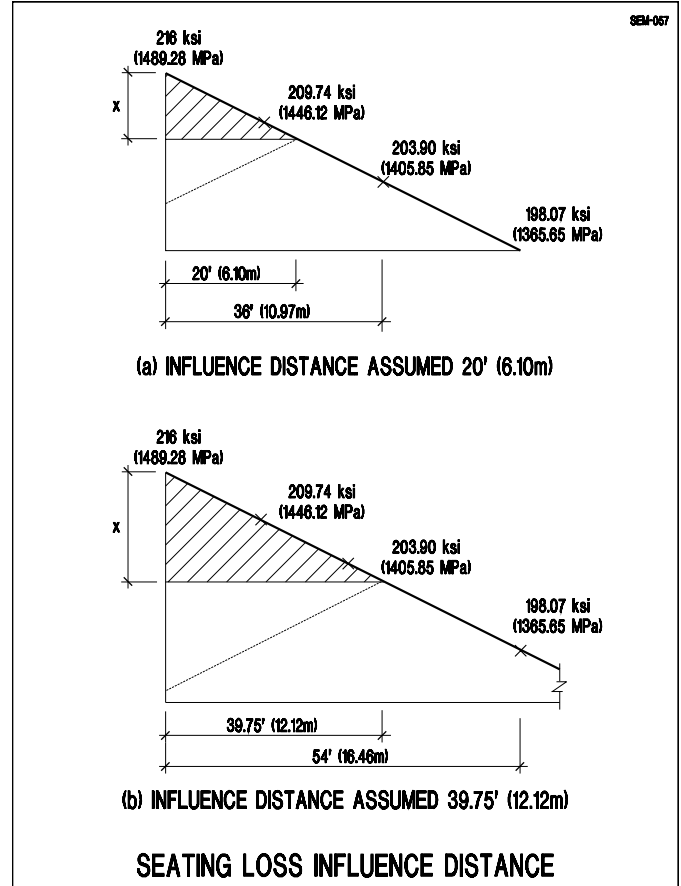
$a = 1 / E_s * \int (final \text{ stress} - initial \text{ stress}) * dx$

where,

$a = \text{anchor set} = 0.25 \text{ in (6.35 mm)}$   
 $E_s = \text{modulus of elasticity of tendon}$   
 $= 28000 \text{ ksi (193054 MPa)}$

Consider the section "x" at 20 ft from the stressing point and calculate the seating loss. (Fig 5.1-2a)

Assume that the stress loss is linear along the span.



**FIGURE 5.1-2**

Consider the section "x" at 39.75 ft from the stressing point and calculate the seating loss. (Fig 5.1-2b)

$(216 - 198.07) / 54 = x / 39.75$   
 $x = 13.20 \text{ ksi}$

Seating loss,  $a = (2 * 13.20 * 39.75 * 12) / (2 * 28000)$   
 $= 0.23 \text{ in (5.59 mm)}$   
 $\approx 0.25 \text{ in}$

Maximum stress in the tendon = 216 - 13.20  
 $= 202.80 \text{ ksi}$

**B. Long-Term Loss**

Long-Term loss calculation for first span:

- Elastic Shortening:

$$ES = K_{es} * (E_s / E_{ci}) * f_{cpa}$$

$$f_{cpa} = 250 \text{ psi (1.72 MPa)}$$

$$E_{ci} = 2440 \text{ ksi (16823 MPa)}$$

$$E_s = 28000 \text{ ksi (193054 MPa)}$$

$$ES = 0.5 * (28000 / 2440) * 0.250$$

$$= 1.434 \text{ ksi (9.887 MPa)}$$

Compressive strength,  $f'_c$   
 $= 27.58 \text{ MPa (4000 psi)}$

Weight  
 $= 2403 \text{ kg/m}^3 \text{ (150 pcf)}$

Modulus of Elasticity  
 $= 24849 \text{ MPa (3604 ksi)}$

■ Shrinkage of Concrete:

$$SH = 8.2 * 10^{-6} * K_{sh} * E_s * [1 - 0.06 * (V/S)] * (100 - RH)$$

$$V/S = 2.50 \text{ in (63.5 mm)}$$

$$RH = 80 \%$$

$$K_{sh} = 0.85 \text{ (from Table 4.3-1 for 3 days)}$$

$$SH = 8.2 * 10^{-6} * 0.85 * 28000 * (1 - 0.06 * 2.5) * (100 - 80)$$

$$= 3.318 \text{ ksi (22.88 MPa)}$$

■ Creep of Concrete:

$$CR = K_{cr} * (E_s / E_c) * f_{cpa}$$

$$E_c = 3604 \text{ ksi (24849 MPa)}$$

$$CR = 1.6 * (28000 / 3604) * 0.250$$

$$= 3.108 \text{ ksi (21.43 MPa)}$$

■ Relaxation of Strands:

$$RE = [K_{re} - J * (SH + CR + ES)] * C$$

For 270 ksi low relaxation strand, from Table 4.4-1:

$$K_{re} = 5 \text{ ksi (34.47 MPa)}$$

$$J = 0.04$$

$$f_{pi} = 213.62 \text{ ksi (1473 MPa)}$$

(considering midspan of first span representatively)

$$f_{pu} = 270 \text{ ksi (1861.60 MPa)}$$

$$f_{pi} / f_{pu} = 213.62 / 270 = 0.79$$

From Table 4.4-2:  $C = 1.22$

$$RE = [5 - 0.04 * (1.434 + 3.318 + 3.108)] * 1.22$$

$$= 5.716 \text{ ksi (39.41 MPa)}$$

$$\text{Total Stress Losses} = 1.434 + 3.318 + 3.108 + 5.716$$

$$= 13.576 \text{ ksi (93.60 MPa)}$$

## 5.2 Friction and Long-Term Stress Loss Calculation of a Post-Tensioned Bonded Beam

### Structure

- Geometry  
Two span beam with grouted post-tensioning as shown in Fig. 5.2-1
- Material Properties
  - o Concrete:

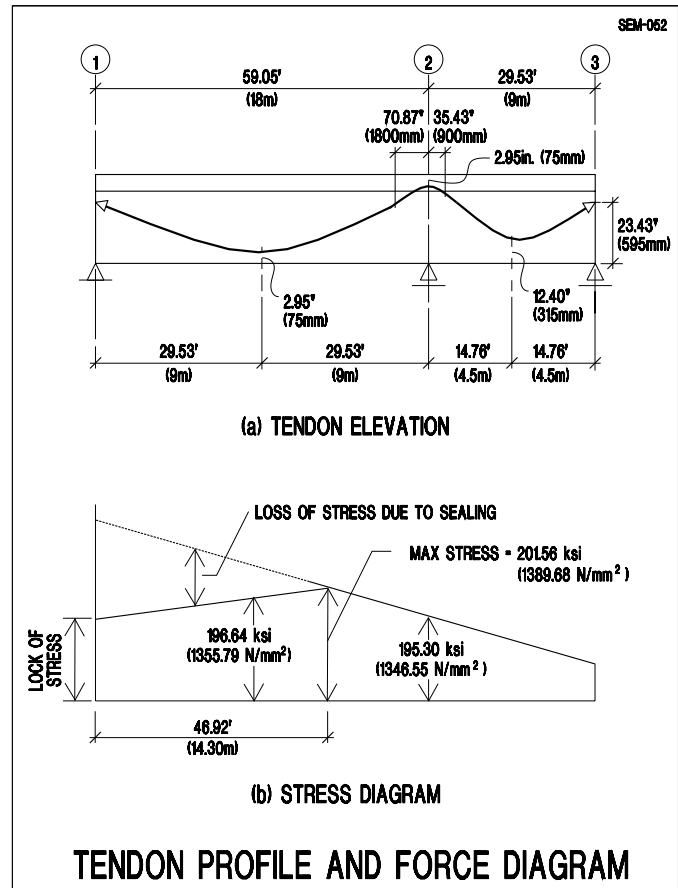


FIGURE 5.2-1

Age of Concrete at stressing  
 $= 3 \text{ days}$

Compressive strength at stressing,  $f'_{ci}$   
 $= 20 \text{ N/mm}^2 \text{ (2901 psi)}$

o Prestressing:

Post-tensioning is provided by a single tendon consisting of 8-13 mm diameter low-relaxation strands stressed at one end

Strand Diameter

$$= 13 \text{ mm (} \frac{1}{2} \text{ in)}$$

Strand Area

$$= 99 \text{ mm}^2 \text{ (0.153 in}^2\text{)}$$

Modulus of Elasticity

$$= 193000 \text{ N/mm}^2 \text{ (27993 ksi)}$$

Coefficient of angular friction,

$$\mu = 0.20$$

Coefficient of wobble friction,

$$K = 0.0002 \text{ rad/m (0.000061 rad/ft)}$$

Ultimate strength of strand,

$$f_{pu} = 1862 \text{ MPa (270 ksi)}$$

Ratio of jacking stress to strand's ultimate strength = 0.8  
Anchor set = 6 mm (0.24 in)

- Loading:  
Selfweight of beam for 6m tributary = 26 kN/m (1.78 k/ft)  
Superimposed sustained load in addition to Selfweight after the beam is placed in service (1.02 kN/m<sup>2</sup>) = 2.63 kN/m (0.18 k/ft)

**Long-Term Loss Calculation :**

The calculation consists of the following steps:

1. Determine the initial stress in tendon at midspan and over second support. Use PT program or hand calculations.
2. Determine the bending moments and stresses at midspan and over the first support due to selfweight and the superimposed sustained loading using ADAPT or other software. Also, calculate stresses due to post-tensioning.
3. Use the relationships given in this document to calculate long-term losses.

The long-hand calculations presented are followed by a printout from the PT for the same example.

**A. Calculation of initial stress in tendon**

The initial tendon stresses ( $f_{pi}$ ) after anchor set may be read off from the Fig 5.2-1 as:

At midspan 1355.79 N/mm<sup>2</sup> (196.64 ksi)  
At support 1346.55 N/mm<sup>2</sup> (195.30 ksi)

**B. Bending moments and stresses at required points**

The sectional properties of the beam are:

Cross sectional area  $A = 720400 \text{ mm}^2$  (1116.62 in<sup>2</sup>)  
Moment of inertia  $I = 5.579 \cdot 10^{10} \text{ mm}^4$  (134036 in<sup>4</sup>)  
Neutral axis to bottom fiber  $Y_b = 595.0 \text{ mm}$  (23.43 in)  
Neutral axis to top fiber  $Y_t = 305.0 \text{ mm}$  (12.01 in)  
Neutral axis to height of strand  
At midspan  $c = 595-75 = 520.0 \text{ mm}$  (20.47 in)  
At support  $c = 305-75 = 230.0 \text{ mm}$  (9.06 in)

The distribution of bending moments due to selfweight of the beam frame from ADAPT-PT or other programs is:

Moment at midspan 658.34 kN-m (485.57 k-ft)  
Moment at support -789.29 kN-m (-582.15 k-ft)

Stresses in concrete at center of gravity of tendons ( $f_g$ ) due to weight of structure at time of stressing are calculated at height of tendon CGS. This is a hypothetical point for concrete, as in the general case there is no concrete at CGS of tendons. The tendon spanning between the supports, and profiled such as to profile an uplift, acts against the weight of the beam on formwork.

$$\begin{aligned} \text{Stress } f_g \text{ at midspan} &= 658.34 \cdot (10^6) \cdot 520 / (5.579 \cdot 10^{10}) \\ &= 6.14 \text{ N/mm}^2 \\ &= (0.891 \text{ ksi})(\text{tension}) \end{aligned}$$

$$\begin{aligned} \text{Stress } f_g \text{ at support} &= 789.29 \cdot (10^6) \cdot 230 / (5.579 \cdot 10^{10}) \\ &= 3.25 \text{ N/mm}^2 \\ &= (0.471 \text{ ksi})(\text{tension}) \end{aligned}$$

Stresses due to superimposed sustained loading  $f_{cds}$  may be prorated from the dead load stresses:

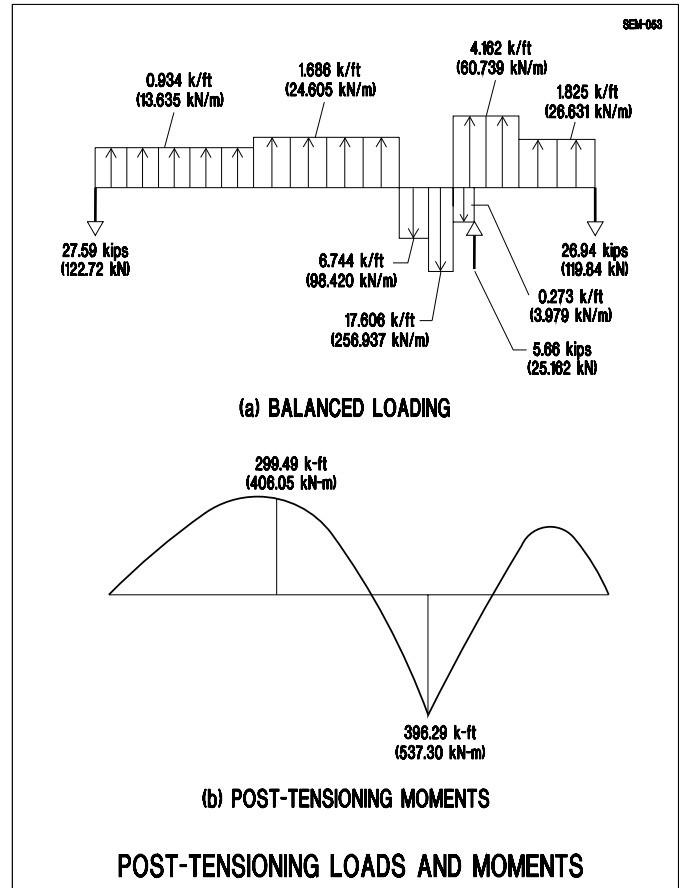


FIGURE 5.2-2

$$\begin{aligned} \text{Stress } f_{cds} \text{ at midspan} &= (2.63/26)*6.14 \\ &= 0.62 \text{ N/mm}^2 \\ &\quad (0.089 \text{ ksi}) \text{ (tension)} \\ \text{Stress } f_{cds} \text{ at support} &= (2.63/26)*3.25 \\ &= 0.33 \text{ N/mm}^2 \\ &\quad (0.048 \text{ ksi}) \text{ (tension)} \end{aligned}$$

The initial stress in concrete is calculated from the balanced loading immediately after the tendon is seated. In other words, before long-term losses take place. The values of the balanced load and the associated post-tensioning moments are given in Fig. 5.2-2.

$$\begin{aligned} \text{At midspan } M_b &= -406.05 \text{ kN-m } (-299.49 \text{ k-ft}) \\ \text{At support } M_b &= 537.30 \text{ kN-m } (396.29 \text{ k-ft}) \end{aligned}$$

Initial concrete stress due to post-tensioning  $f_{cpi}$ :  
At midspan:

$$\begin{aligned} \text{The initial post-tensioning force} \\ P_{pi} &= 8*98*1355.79/1000 \\ &= 1062.94 \text{ kN } (238.96 \text{ kips}) \\ f_{cpi} &= P_{pi}/A + M_b*c/I \\ &= 1062.94*10^3/720400 + 406.05*(10^6)*520/ \\ &\quad (5.579*10^{10}) \\ &= 5.26 \text{ N/mm}^2 \text{ (763 psi) (C)} \end{aligned}$$

At support:

$$\begin{aligned} \text{The initial post-tensioning force} \\ P_{pi} &= 8*98*1346.55/1000 \\ &= 1055.70 \text{ kN } (237.33 \text{ kips}) \\ f_{cpi} &= 1055.70*10^3/720400 + 537.30*(10^6)*230/ \\ &\quad (5.579*10^{10}) \\ &= 3.68 \text{ N/mm}^2 \text{ (534 psi) (C)} \end{aligned}$$

### C. Calculation of long-term stress losses

#### i. At Mid-span

- Elastic shortening:

$$\begin{aligned} ES &= K_{es}*E_s*f_{cir}/E_{ci} \\ K_{es} &= 0 \text{ (all strands are pulled and} \\ &\quad \text{anchored simultaneously)} \end{aligned}$$

Hence,

$$ES = 0 \text{ psi}$$

It is observed that, in this example, the long-term losses due to elastic shortening are zero since all the strands are stressed and anchored simultaneously.

- Creep of concrete:

For the calculation of losses due to creep, the initial stress in concrete  $f_{cir}$  will be calculated with both the selfweight and the sustained superimposed loadings considered as active. Hence,

$$\begin{aligned} f_{cir} &= 0.88 \text{ N/mm}^2 \text{ (128 psi) (T)} \\ &\quad \text{from elastic shortening} \\ &\quad \text{calculations} \\ f_{cds} &= 0.62 \text{ N/mm}^2 \text{ (90 psi) (T) from} \\ &\quad \text{stress calculations} \end{aligned}$$

$$CR = K_{cr}*(E_s/E_c)*(f_{cir} - f_{cds})$$

$$\begin{aligned} K_{cr} &= 1.6 \\ E_c &= 4700*(27.58)^{1/2} \\ &= 24683 \text{ N/mm}^2 \text{ (3580 ksi)} \end{aligned}$$

$$\begin{aligned} (f_{cir} - f_{cds}) &= -0.88 - 0.62 \\ &= -1.50 \text{ N/mm}^2 \text{ (-218 psi) (T)} \end{aligned}$$

It is observed that the net stresses ( $f_{cir} - f_{cds}$ ) are tensile. Stress loss due to creep is associated with compressive stresses only. A negative sum is substituted by zero. Therefore,

$$\begin{aligned} CR &= 1.6*(193000/24683)*0.0 \\ &= 0 \text{ N/mm}^2 \end{aligned}$$

- Shrinkage of concrete

The relationship used for shrinkage is:

$$\begin{aligned} SH &= 8.2*10^{-6}*K_{sh}*E_s*(1 - \\ &\quad 0.0236*V/S)*(100 - RH) \end{aligned}$$

Where,

$$\begin{aligned} K_{sh} &= 0.85 \text{ (for stressing at 3 days -} \\ &\quad \text{Table 4.3-1)} \\ RH &= 0.70 \text{ (given relative humidity)} \end{aligned}$$

$$\begin{aligned} V/S &= \text{volume to surface ratio} \\ &= (2580*130 + 500*770)/(2*2580 \\ &\quad + 2*770) \\ &= 107.52 \text{ mm } (4.23 \text{ in}) \end{aligned}$$

$$\begin{aligned} SH &= 8.2*10^{-6}*0.85*193000* \\ &\quad (1 - 0.0236*107.52)*(100 - 70) \\ &= 30.12 \text{ N/mm}^2 \text{ (4.369 ksi)} \end{aligned}$$

- Relaxation of tendon

$$RE = (K_{re} - J*(SH + CR + ES))*C$$

$$f_{pi} = 1355.79 \text{ N/mm}^2 (196.64 \text{ ksi})$$

$$f_{pi}/f_{pu} = 1355.79/1862 = 0.73$$

$$C = 0.90 \text{ (from Table 4.4-2)}$$

$$K_{re} = 34.47 \text{ N/mm}^2 (5000 \text{ psi});$$

$$J = 0.04 \text{ (from Table 4.4-1)}$$

$$RE = [34.47 - 0.04*(0 + 30.12 + 0)]*0.90$$

$$= 29.94 \text{ N/mm}^2 (4.342 \text{ ksi})$$

Hence, total stress loss is given by:

$$TL = 0 + 30.12 + 0 + 29.94$$

$$= 60.06 \text{ N/mm}^2 (8.711 \text{ ksi})$$

## ii. At Second Support

Over the second support the stress losses are computed as follows:

- Elastic shortening :

$$f_{cir} = 1.0*3.68 - 3.25$$

$$= 0.43 \text{ N/mm}^2 (62 \text{ psi}) (C)$$

$$ES = 0.5*(193000/21019)*0.43$$

$$= 1.97 \text{ N/mm}^2 (0.286 \text{ ksi})$$

- Shrinkage :

Same as in the preceding case;

$$SH = 30.12 \text{ N/mm}^2 (4.369 \text{ ksi})$$

- Creep

$$f_{cir} = 0.43 \text{ N/mm}^2 (64 \text{ psi}) (C)$$

$$f_{cds} = 0.33 \text{ N/mm}^2 (48 \text{ psi}) (T)$$

$$CR = 1.60*(193000/24683)*(0.43 - 0.33)/1000$$

$$= 0.001 \text{ N/mm}^2 (0 \text{ ksi})$$

- Relaxation

$$f_{pi} = 1346.55 \text{ N/mm}^2 (195.30 \text{ ksi})$$

$$f_{pi}/f_{pu} = 1346.55/1862 = 0.73$$

$$C = 0.90$$

$$RE = [34.47 - 0.04*(30.12 + .001 + 1.97)]*0.90$$

$$= 29.87 \text{ N/mm}^2 (4.334 \text{ ksi})$$

Hence, total stress loss is

$$TL = 30.12 + 0.001 + 1.97 + 29.87$$

$$= 61.96 \text{ N/mm}^2 (8.987 \text{ ksi})$$

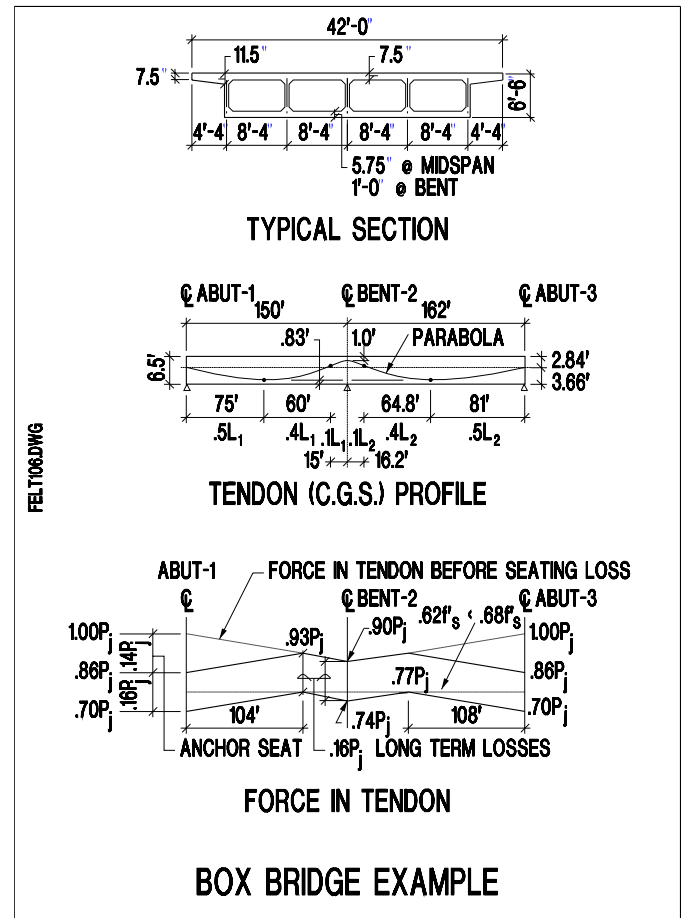


FIGURE 5.3-1

## 5.3 Friction Loss Calculation of a Post-Tensioned Box Girder Bridge

Consider the Box girder bridge shown in Fig. 5.3-1.

### Sample Calculation Of Stress in Tendon after Seating Loss

At Second Support:

Stress at distance x from the jacking point,

$$P_x = P_s * e^{-(\mu\alpha + Kx)}$$

$$P_s = \text{Stress at jacking point} = P_j$$

$$\mu = \text{Coefficient of angular friction}$$

$$= 0.25/\text{radian}$$

$$K = \text{Coefficient of wobble friction}$$

$$= 0.0002 \text{ rad/ft} (0.00066 \text{ rad/m})$$

$$x = \text{distance from the stressing point}$$

$$= 150 \text{ ft} (45.72 \text{ m})$$

$$\alpha = \text{Change of angle in strand (radians) from the stressing point to distance x}$$

$$= 4*e/1 + 2*e/1 = 4*(3.66 - 0.83)/135 +$$

$$2*(2.84 - 1)/15$$

$$= 0.329 \text{ rad}$$

$$P_x = P_s * e^{-(\mu\alpha + Kx)} = P_j * e^{-(0.25 * 0.329 + 0.0002 * 150)}$$

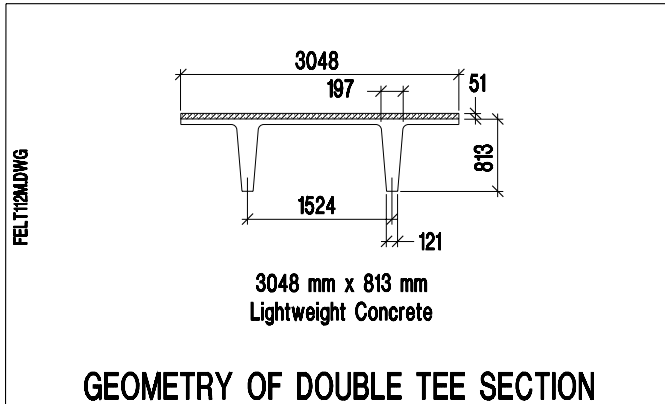
$$= 0.89 P_j$$

## 5.4 Long-Term Stress Loss Calculation of a Precast Beam

Consider the precast double tee section shown in **Figure 5.4-1** with a 51 mm topping.

### Structure

Geometry and section properties are shown in **Figure 5.4-1** & Table 5.4-1



**FIGURE 5.4-1**

Table 5.4-1 Section Properties

$f'_c$	= 34.47 N/mm <sup>2</sup> (5 ksi)
$f_{pu}$	= 1861 N/mm <sup>2</sup> (270 ksi)
Span	= 21.34 m (70 ft)
A	= 396773 mm <sup>2</sup> (615 in <sup>2</sup> )
I	= 2.49*10 <sup>10</sup> mm <sup>4</sup> (59.82*10 <sup>3</sup> in <sup>4</sup> )
$Z_b$	= 4.45*10 <sup>7</sup> mm <sup>3</sup> (2715.55 in <sup>3</sup> )
V/S	= 396773/9245.6 = 42.91 mm (1.70 in)
weight	= 7.166 kN/m (0.491 k/ft)
weight of topping	= 3.649 kN/m (0.250 k/ft)

- Material Properties

- o Concrete:

Compressive strength,  $f'_c = 34.47$  MPa  
(5000 psi)

Weight = 2403 kg/m<sup>3</sup> (150 pcf)

Modulus of Elasticity,

$$E_c = 4700 * (34.47)^{1/2} \\ = 27594 \text{ N/mm}^2 \quad (4002 \text{ ksi})$$

Compressive strength at stressing,

$$f'_{ci} = 24.13 \text{ N/mm}^2 \quad (3500 \text{ psi})$$

Modulus of Elasticity,

$$E_{ci} = 4700 * (24.13)^{1/2} \\ = 23087 \text{ N/mm}^2 \quad (3349 \text{ ksi})$$

- o Prestressing:

Prestressing is provided by 12-11 mm diameter stress relieved strands depressed at midspan with following eccentricities:

$$e_c = 475.7 \text{ mm (18.73 in)}$$

$$e_e = 325.4 \text{ mm (12.81 in)}$$

$$\text{Strand Diameter} = 11 \text{ mm (0.4 in)}$$

$$\text{Modulus of Elasticity} = 193000 \text{ N/mm}^2 \\ (27993 \text{ ksi})$$

Ultimate strength of strand,

$$f_{pu} = 1861 \text{ MPa (270 ksi)}$$

Assume ambient humidity

$$H = 70\%$$

Tendons are stressed to  $0.7 * f_{pu}$ ; hence,

$$f_{pi} = 0.7 * 1861 \\ = 1302.7 \text{ N/mm}^2 \quad (188.94 \text{ ksi})$$

- Loading:

Selfweight (lightweight concrete)  
= 7166 N/m (0.491 k/ft)

Superimposed sustained loading (51 mm topping)  
= 3649 N/m (0.250 k/ft)

### Required:

Determine the total long-term stress losses at the critical point.

### Calculation:

In a simply supported beam with a straight tendon depressed at midspan, the critical stress location is generally near the 0.4 of span. The moment at 0.4L is given by

$$M = 0.4L * (wL/2) - w * (0.4L)^2 / 2 \\ = 0.12 * w * L^2$$

$$M_d = 0.12 * 7.166 * 21.34^2 \\ = 391.6 \text{ kN-m (288.83 k-ft)}$$

$$M_{eds} = 0.12 * 3.649 * 21.34^2 \\ = 199.4 \text{ kN-m (147.07 k-ft)}$$

Eccentricity at 0.4L is:

$$e = 325.4 + 0.8 * (475.7 - 325.4) \\ = 445.6 \text{ mm (17.54 in)}$$

$$P_i = 0.7 * A_{ps} * f_{pu} \\ = 0.7 * 12 * 95.63 * 1861 \\ = 1495 \text{ kN (336.09 kip)}$$

Stress due to selfweight  $f_g$ :

$$f_g = 391.6 * (10^6) * 445.6 / (2.49 * 10^{10}) \\ = 7.01 \text{ N/mm}^2 \quad (1.017 \text{ ksi}) \quad (\text{T, tension})$$

Stress due to superimposed sustained loading  $f_{eds}$ :

$$f_{eds} = 199.4 * (10^6) * 445.6 / (2.49 * 10^{10}) \\ = 3.57 \text{ N/mm}^2 \quad (0.518 \text{ ksi}) \quad (\text{T, tension})$$

Stress due to prestressing  $f_{cpi}$ :

$$\begin{aligned} f_{cpi} &= P_i/A + P_i * e^2/I \\ &= (1.495 * 10^6)/396773 + \\ &\quad (1.495 * 10^6) * 445.6^2 / (2.49 * 10^{10}) \\ &= 15.69 \text{ N/mm}^2 (2.276 \text{ ksi}) (C) \end{aligned}$$

**A. Elastic shortening:**

$$ES = K_{es} * E_s * f_{cir} / E_{ci}$$

where,

$$\begin{aligned} K_{es} &= 1 \\ f_{cir} &= K_{cir} * f_{cpi} - f_g, \text{ with} \\ K_{cir} &= 0.9 \text{ for pretensioning} \\ f_{cir} &= 0.9 * 15.69 - 7.01 \\ &= 7.11 \text{ N/mm}^2 (1.031 \text{ ksi}) (C) \end{aligned}$$

$$\begin{aligned} ES &= 1 * 193000 * 7.11 / 23087 \\ &= 59.44 \text{ N/mm}^2 (8.621 \text{ ksi}) \end{aligned}$$

**B. Creep of concrete:**

$$CR = K_{cr} * (E_s/E_c) * (f_{cir} - f_{cds})$$

where,

$$K_{cr} = 2.0 \text{ for pretensioned members; reduce 20\% due to lightweight concrete.}$$

$$\begin{aligned} CR &= 2.0 * 0.8 (193000 / 27594) * \\ &\quad (7.11 - 3.57) \\ &= 39.62 \text{ N/mm}^2 (5.746 \text{ ksi}) (C) \end{aligned}$$

**C. Shrinkage of concrete:**

$$\begin{aligned} SH &= 8.2 * 10^{-6} * K_{sh} * E_s * \\ &\quad (1 - 0.00236 * V/S) * (100 - RH) \end{aligned}$$

where,

$$K_{sh} = 1 \text{ for pretensioned members}$$

$$\begin{aligned} SH &= 8.2 * 10^{-6} * 1 * 193000 (1 - \\ &\quad 0.00236 * 42.91) * (100 - 70) \\ &= 42.67 \text{ N/mm}^2 (6.189 \text{ ksi}) \end{aligned}$$

**D. Relaxation of strands:**

$$RE = [K_{re} - J(SH + CR + ES)] * C$$

$$\begin{aligned} K_{re} &= 137.90 \text{ N/mm}^2 (20 \text{ ksi}); \\ J &= 0.15 \text{ (Table 4.4-1)} \end{aligned}$$

$$\begin{aligned} f_{pi} / f_{pu} &= 0.7; \text{ hence,} \\ C &= 1.00 \text{ (Table 4.4-2)} \end{aligned}$$

$$\begin{aligned} RE &= [137.90 - 0.15 * (42.67 + 39.62 + \\ &\quad 59.44)] * 1 \\ &= 116.64 \text{ N/mm}^2 (16.917 \text{ ksi}) \end{aligned}$$

**E. Total long-term loss:**

$$\begin{aligned} LT &= ES + CR + SH + RE \\ &= 59.44 + 39.62 + 42.67 + 116.64 \\ &= 258.37 \text{ N/mm}^2 (37.474 \text{ ksi}) \end{aligned}$$

$$\begin{aligned} \text{Final prestress} &= 0.7 * 1861 - 258.37 \\ &= 1044.33 \text{ N/mm}^2 (151.470 \text{ ksi}) \end{aligned}$$

## NOTATION

a = Anchor set;  
A = cross sectional area;  
CR = stress loss due to creep;  
e = eccentricity of tendon from centroidal axis;  
E<sub>c</sub> = concrete's modulus of elasticity at 28 days;  
E<sub>ci</sub> = concrete's modulus of elasticity at stressing age;  
ES = stress loss due to elastic shortening;  
E<sub>s</sub> = strand's modulus of elasticity;  
f<sub>cds</sub> = stress in concrete at center of gravity of tendons due to all superimposed permanent dead loads that are applied to the member after it has been prestressed;  
f<sub>cir</sub> = net stress in concrete at center of gravity of tendons immediately after prestress has been applied to concrete;  
f<sub>cpa</sub> = average compressive stress in concrete immediately after stressing, at a hypothetical location defined by the center of gravity of tendons;  
f<sub>pi</sub> = stress in tendon immediately after transfer of prestressing;  
f<sub>pu</sub> = ultimate strength of strand;  
I = moment of inertia;  
J = a coefficient for stress relaxation in tendon (**Table 4.4-1**);  
K = wobble coefficient of friction expressed per unit length of strand;  
K<sub>cir</sub> = an adjustment coefficient for loss due to elastic shortening;  
K<sub>cr</sub> = creep coefficient;  
K<sub>es</sub> = a coefficient for elastic shortening stress loss calculation;  
K<sub>re</sub> = a coefficient for stress relaxation in tendon (**Table 4.4-1**);  
K<sub>sh</sub> = a shrinkage constant (**Table 4.3-1**);  
M = moment;  
P<sub>x</sub> = stress at distance x from the jacking point;  
RE = stress loss due to relaxation of tendon;  
RH = relative humidity (percent);  
SH = stress loss due to shrinkage of concrete;

V/S = volume to surface ratio;  
Y<sub>b</sub> = centroidal axis to bottom fiber;  
Y<sub>t</sub> = centroidal axis to top fiber;  
α = change of angle in strand (radians) from the stressing point to distance X; and,  
μ = coefficient of angular friction.

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1733 Woodside Road, Suite 220, Redwood City, CA 94061, USA TEL 650.306.2400 FAX 650.364.4678 E-MAIL info@adaptsoft.com www.adaptsoft.com