# DISTRIBUTION OF STRESS IN GROUND-SUPPORTED SLABS

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This Technical Note describes the distribution of stress in ground-supported slabs due to loads applied at the slab edges. It covers un-reinforced, conventionally reinforced and post-tensioned slabs. The Note concludes that at locations away from the slab boundary, the distribution of stress due to selfweight and post-tensioning is uniform and axial. This conclusion is based on the fact that in ground-supported slabs, the distribution of stress is primarily governed by *changes* in the profile of the underlain soil.

The material presented in this Technical Note can readily be verified using a two-dimensional finite element analysis that models a section through the slab. However, the discussion is kept within the confines of simple beam theory, and common engineering knowledge, in order to reach a wider audience of the community interested in the behavior of ground-supported slabs.

Consider the slab shown in **Fig. 1**. The slab is assumed to be long (a length-to-thickness ratio of 15 or more) and have a uniform thickness. The soil support is assumed rigid. In practice, soil is flexible. The impact of soil flexibility on the distribution of stress is addressed in the latter part of the discussion.
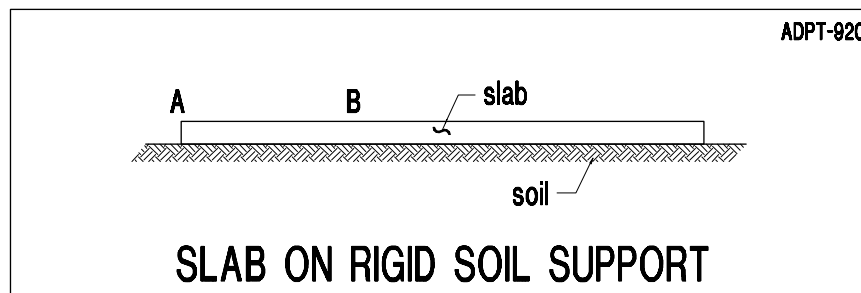
FIGURE 1

When concrete is cast, it assumes the profile of the underlain soil. Before concrete sets, its weight is directly transferred to the soil below it and is uniformly distributed.

DISTRIBUTION OF STRESS
Using simple beam theory, the moment generated in a slab (M) is given by the following relationship.

$$M = (EI)/R$$

Where,

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E   = modulus of elasticity of slab;
I   = second moment of area of slab; and
R   = change in radius of curvature of the slab.

The above relationship is based on the assumption that plane sections remain plane. It applies to regions not in proximity [2] to discontinuities, or point of application of concentrated loads, such as a slab edge.

At the onset, the curvature in the slab will be the same as that of its supporting soil. The bending stresses generated in a slab, subsequent to setting of concrete, is determined by the *change* in the curvature of the underlain soil, that is to say, change in the profile of the slab after it sets. If an applied load does not result in a *change* in profile of the soil, on the assumption that the slab maintains contact with soil, there will be no change in (R), and hence no bending stresses in the slab.

To illustrate the concept, several simple scenarios will be presented. Post-tensioned slabs are also discussed. In all the examples, it is assumed that the load is distributed along the edge of the slab at a pre-defined distance from slab centroid given in each example.

Consider the case of the hypothetical slab supported on rollers and rigid supports, and subjected to an externally applied axial load P at its edge (**Fig. 2-a**).   At location B, away from the slab edge, the entire axial force P will be available, with the stress being uniformly distributed over the slab thickness (**Fig. 2-b**).

In **Fig. 2-c**, the force P is applied with an eccentricity "e" with respect to the centroid of the slab. This is equivalent to a moment (P*e) and an axial force (P) at the slab edge. The moment component will be neutralized through adjustment in the soil pressure over a short length from the end of the slab.

Away from the slab edge, there will be no change in the profile of the slab support–hence no moments. Consequently, distribution of stress in the slab will be uniform compression (**Fig. 2-d**).

Figure 3-a considers the case where friction between the slab and the underlain soil over the distance AB dissipates a portion of the axial compression. In this scenario, friction develops at the point where the slab retains its contact with the soil. Bending stresses in the slab will be generated if the profile of the soil support is changed. Reiterating the premise that the moment in the slab is a function of the *change* in the profile of the soil, no moment will develop if the soil profile does not change. The resultant force at B will be (P-F), where F is the friction between A and B. The force will act at the centroid of the slab, resulting in a uniform distribution of stress[3].

The above scenarios lead to the loading condition of a post-tensioned slab on rigid support (**Fig. 3-c**). Using similar arguments, for an eccentrically stressed post-tensioned slab (**Fig. 3-d**), the distribution of stress at section B will be uniform compression.

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[2] A distance two to three times the slab thickness is generally considered far enough.
[3] Strictly speaking, there will be a small change is the distribution of axial force at section B, effected by the friction forces at the immediate regions on each side of section B. The change is not significant. It is not governed by the simple beam theory, and is discusses later in the Technical Note.
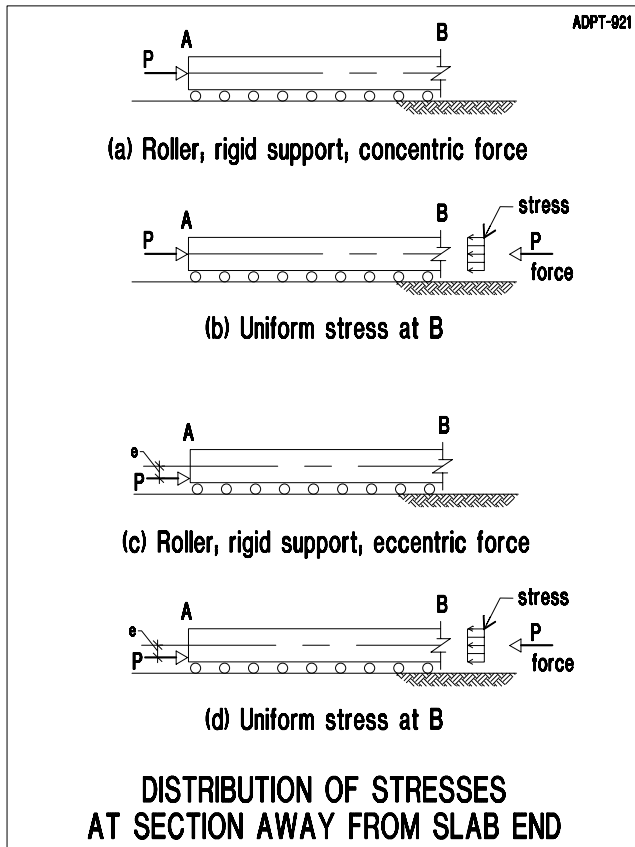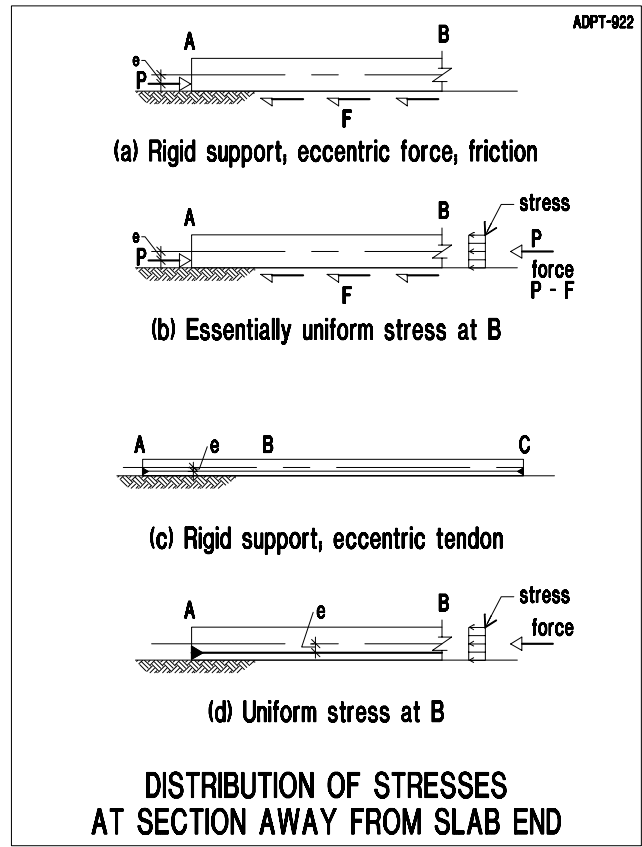
(a) Roller, rigid support, concentric force

(b) Uniform stress at B

(c) Roller, rigid support, eccentric force

(d) Uniform stress at B

DISTRIBUTION OF STRESSES
AT SECTION AWAY FROM SLAB END

FIGURE 2

(a) Rigid support, eccentric force, friction

(b) Essentially uniform stress at B

(c) Rigid support, eccentric tendon

(d) Uniform stress at B

DISTRIBUTION OF STRESSES
AT SECTION AWAY FROM SLAB END

FIGURE 3

FREE BODY DIAGRAMS FOR RIGID SUPPORTS
The statements made regarding uniform distribution of stress at a section located a short distance away from a slab edge can be explained through the examination of the free body diagram of ground-supported slabs. Two cases are considered, one with the post-tensioning tendon below the centroid of the slab and one above it.

Figure 4-a is the illustration of a post-tensioned ground-supported slab with tendons located below the slab centroid. The eccentricity moment due to post-tensioning is assumed to be large enough to raise the slab ends. Part b of the figure shows the deflected shape of the slab. The forces acting on the slab are shown in **Fig. 4-c**. The concentrated force (R) generated at the edge of the slab will be counteracted by the weight of the slab. At distance "a" from the slab edge, both the moment and the shear in the slab will be zero (parts d and e in the figure). Beyond distance "a," the distribution of stress in the slab will be a uniform compression (**Fig. 4-f**).
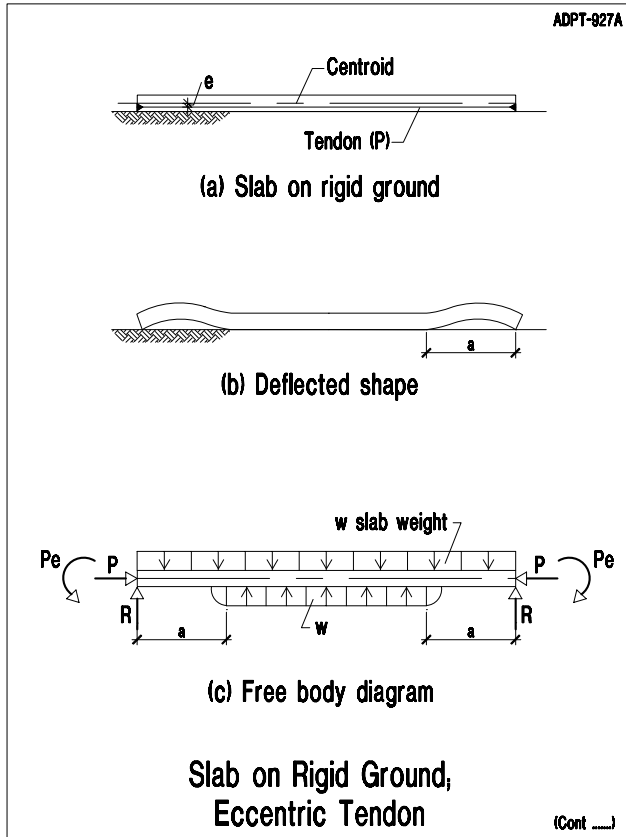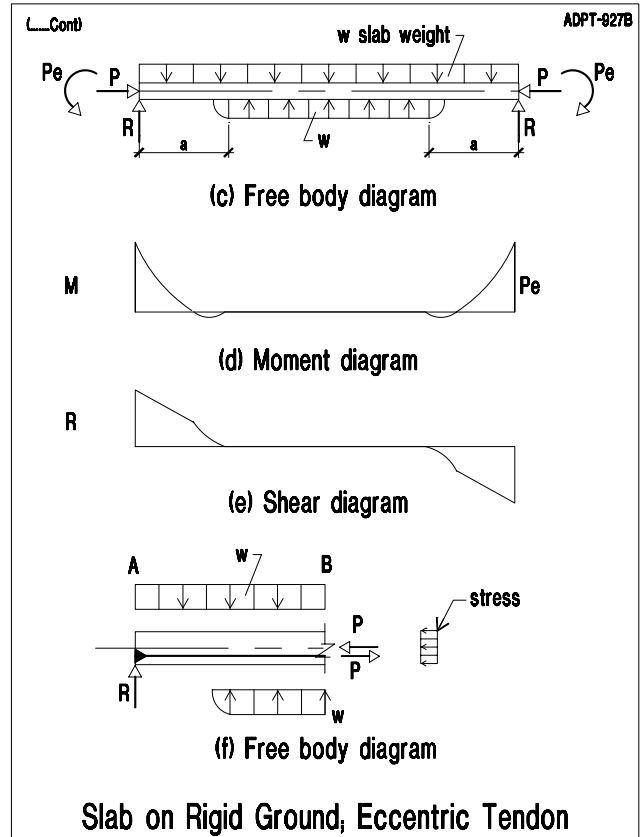
FIGURE 4A

FIGURE 4B

Where tendon eccentricity is above the slab centroid, the slab edge tends to curl up (**Fig. 5-b**). Depending on the magnitude of the force and its eccentricity, the force distribution in the slab is likely to be as shown in **Fig. 5-c**. The moment and shear at distance "a" from the slab edge will be zero (**Fig. 5 d and e**). As a result, the distribution of forces over the concrete section at the regions to the right of distance "a" will be a uniform compression.

FREE BODY DIAGRAMS FOR FLEXIBLE SUPPORTS
Real soil foundations are not rigid. There will be an initial deformation due to the weight of wet concrete. This will not result in bending stresses in the slab, since the immediate change in profile of the soil takes place before concrete sets.

When a force is applied at a slab edge, either as an external force, or from a post-tensioned tendon, the downward, or upward deformation caused at the end region of the slab will cause a local change in soil profile. The local change in slab profile smoothens the distribution of forces illustrated in **Figs. 4** and **5**. The free body diagram for the real soil conditions is likely to be in form of distributions shown in **Fig 6**. In the two cases shown in the figure, again at regions away from the slab edge, the soil surface remains essentially unaffected. Hence the stress distribution will be uniform compression.
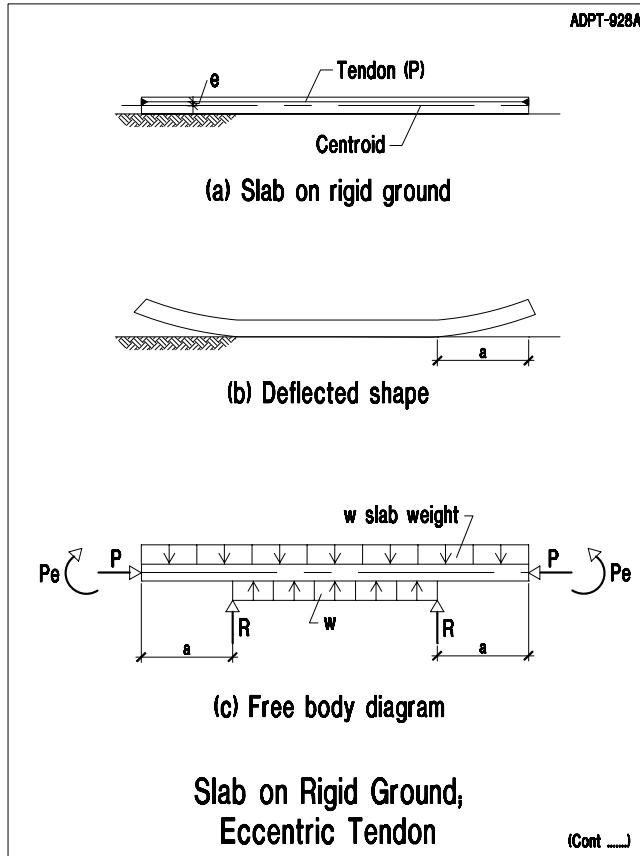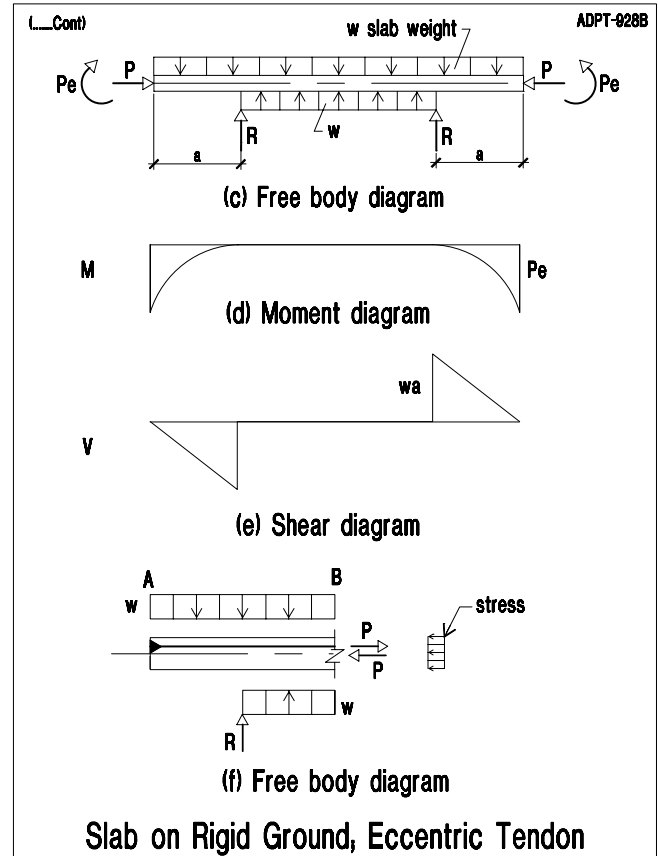
FIGURE 5A

FIGURE 5B

IMPACT OF FRICTION ON DISTRIBUTION OF STRESS

Invariably, there will be some friction at the soil/slab interface. In theory, if a slab is very long, at a distance far enough from the slab edge, friction fully exhausts the influence of a force applied at the slab edge - be the force from a post-tensioned tendon, or applied externally. where friction forces exist, their impact is (i) a reduction in the axial force; (ii) a minor modification in the distribution of the otherwise uniform axial stress across the slab depth. The deviation in the uniform distribution of axial stress over the cross section is due to the influence of frictional forces at the immediate vicinity of the point being considered. Frictional forces that are farther away do not impact the shape of the uniform distribution of axial stresses. This will be explained by way of several free body diagrams that follow.

The free body diagrams presented are attempts to follow common engineering knowledge and concepts of simple beam theory. Obviously, a rigorous analysis would explain the results more conclusively, but such an analysis is beyond the scope of this document. **Figure 7a** shows a long post-tensioned slab on rigid soil foundation. The remainder of the models in the figure are selected with the objective of illustrating that the distribution of stress away from the slab end is uniform compression, adjusted by the impact of local friction. The "local friction" is typically the frictional forces acting over a short distance on each side of the location being considered. In the case of a long slab, at any point away from the slab edge, the otherwise uniform compression is influenced by the friction forces over a length of approximately four times the slab thickness. Since four times the slab thickness is generally a small fraction of the slab length, the impact of friction in the shape of stress distribution is generally insignificant.
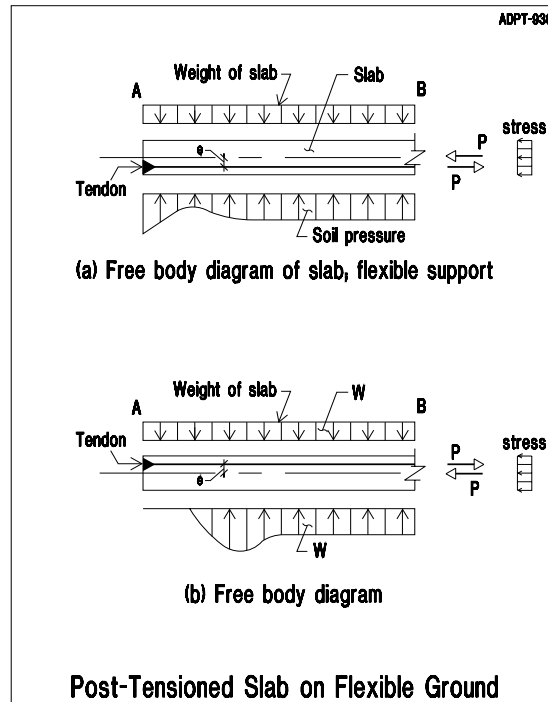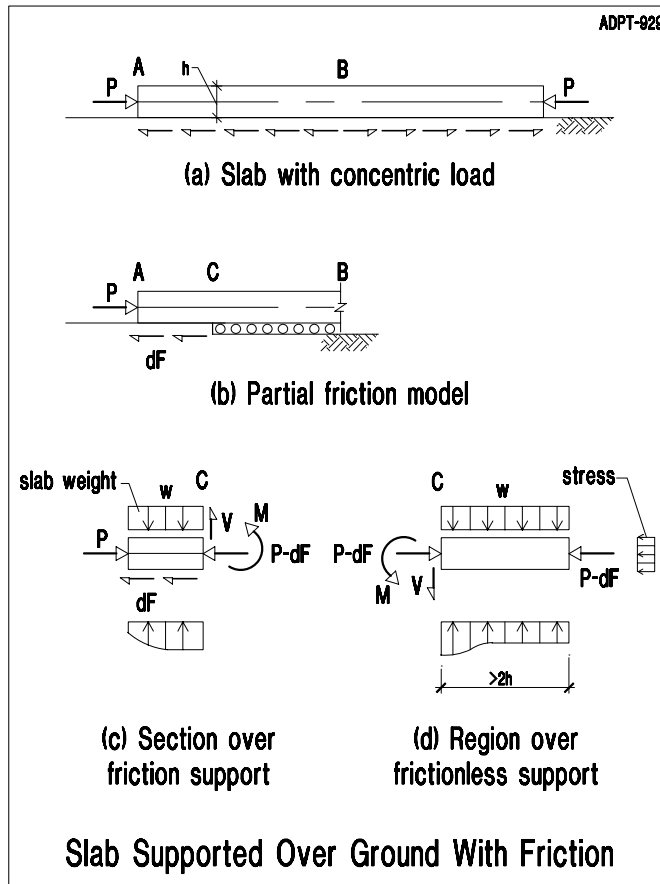
FIGURE 6

Figure 7b shows the free body diagram of a slab segment between the slab end A and section C. The moment at section C is due to the friction force dF minus the moment due to the vertical forces. Note that the tendency of the slab to curve is counteracted by the redistribution of slab weight on the soil. Also note that the distribution of axial stresses due to M is not governed by the simple beam theory (M/I = f/c)[4], since there is no curvature in the slab.

Next, consider the distribution of the actions M, V and the net axial force (P – dF) on the remainder of the slab away from section C, assuming no friction to the right of section C. Using the arguments of prior examples regarding no change in curvature of soil to the right of section C, the stresses away from section C will be uniform compression (part c) of the figure. The impact of a friction force at a given point on the distribution shape of axial stresses in a slab tapers off with distance from the point under consideration. The predominant action at any given section in the slab is the axial force and its uniform distribution on the section.

The above conclusion is valid, regardless of whether the tendon is concentric or eccentric. As illustrated in earlier examples, the impact of tendon eccentricity is confined to changes in the distribution of stress near the slab edge . At regions away from the slab edge the distribution of stress due to prestressing is uniform.

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[4] M = moment; I = second moment of area; f = bending stress due to M; and c = distance from centroid of section

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FIGURE 7